Writing Polynomial Equations from Graphs: Roots, Multiplicity, and End Behavior

Worked example / complete source key

A polynomial graph crosses the x-axis at x = -2, touches at x = 1, and crosses at x = 3. Both ends fall. The graph passes through (0, 6). Write a factored equation and state the degree.

Answer: f(x) = -(x + 2)(x - 1)^2(x - 3), degree 4.

Reason: Cross at -2 gives factor (x + 2) with multiplicity 1. Touch at 1 gives (x - 1)^2 with multiplicity 2. Cross at 3 gives (x - 3) with multiplicity 1. Sum of multiplicities is 1 + 2 + 1 = 4, which is even, matching the end behavior that both ends fall. Write f(x) = a(x + 2)(x - 1)^2(x - 3). Substitute (0, 6): 6 = a(0 + 2)(0 - 1)^2(0 - 3) = a(2)(1)(-3) = -6a, so a = -1. Thus f(x) = -(x + 2)(x - 1)^2(x - 3). The degree is 4, and the negative leading coefficient with even degree makes both ends fall, consistent with the graph.

Check: Substitute x = 0 into f(x) = -(x + 2)(x - 1)^2(x - 3): -(2)(1)(-3) = 6, matching the given point. Confirm the degree is 4 (even) and the leading coefficient is negative, so both ends fall.