Writing Polynomial Equations from Graphs: Roots, Multiplicity, and End Behavior

Original tasks 1, 2, 3 / complete keys

Task 1

A polynomial graph crosses the x-axis at x = -1 and x = 4, and touches at x = 2. Both ends fall. The graph passes through (0, 8). Write a factored equation and state the degree.

Answer: f(x) = -(1/2)(x + 1)(x - 2)^2(x - 4), degree 4.

Reason: Cross at -1 gives (x + 1); touch at 2 gives (x - 2)^2; cross at 4 gives (x - 4). Sum of multiplicities is 1 + 2 + 1 = 4, even, matching both ends falling. Substitute (0, 8): 8 = a(1)(4)(-4) = -16a, so a = -1/2. The leading coefficient is negative, consistent with both ends falling. Degree 4.

Check: Substitute x = 0 and confirm the output is 8; confirm degree 4 is even and the leading coefficient is negative so both ends fall.

Task 2

A polynomial graph crosses at x = -3 with a flattening shape, crosses at x = 0, and crosses at x = 5. The left end falls and the right end rises. Write a factored equation with leading coefficient 1 and state the degree.

Answer: f(x) = (x + 3)^3 x (x - 5), degree 5.

Reason: Flattening cross at -3 suggests multiplicity 3. Cross at 0 gives x^1. Cross at 5 gives (x - 5)^1. Sum = 3 + 1 + 1 = 5, odd, matching opposite end behavior. Right end rises means positive leading coefficient, so a = 1. Degree 5.

Check: Check that the sum of exponents is 5 and that the right end rises for positive leading coefficient; verify intercepts at -3, 0, 5.

Task 3

A polynomial graph touches at x = -2 and touches at x = 3. Both ends fall. The graph passes through (0, -36). Write a factored equation and state the degree.

Answer: f(x) = -(x + 2)^2(x - 3)^2, degree 4.

Reason: Touch at -2 gives (x + 2)^2; touch at 3 gives (x - 3)^2. Sum = 4, even, matching both ends falling. Both ends fall means negative leading coefficient. Substitute (0, -36): -36 = a(2)^2(-3)^2 = a(4)(9) = 36a, so a = -1. Degree 4.

Check: Substitute x = 0 to confirm -36; confirm even degree and negative leading coefficient produce both ends falling.

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Original tasks 4 / complete keys

Task 4

A polynomial graph crosses at x = -4, touches at x = -1, and crosses at x = 2. Both ends fall. The graph passes through (0, 4). Write a factored equation and state the degree.

Answer: f(x) = -(1/2)(x + 4)(x + 1)^2(x - 2), degree 4.

Reason: Cross at -4 gives (x + 4); touch at -1 gives (x + 1)^2; cross at 2 gives (x - 2). Sum = 1 + 2 + 1 = 4, even, matching both ends falling. Both ends fall means negative leading coefficient. Substitute (0, 4): 4 = a(4)(1)(-2) = -8a, so a = -1/2. Degree 4.

Check: Substitute x = 0 to confirm 4; confirm even degree and negative leading coefficient give both ends falling.

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Source rubric / reflection

- [ ] Correctly lists all x-intercepts from the graph.
- [ ] Assigns odd multiplicity to crossings and even multiplicity to touches.
- [ ] Uses end behavior to determine degree parity and leading coefficient sign.
- [ ] Writes a factored equation with correct factors and exponents.
- [ ] Determines the leading coefficient using a test point when needed.
- [ ] States the degree and verifies consistency with end behavior.
- [ ] Expands correctly only if requested.

Reflection: What did you check, what did you change, and what will you try next?