Graph Rational Functions: Vertical and Horizontal Asymptotes

Complete worked example / separate answer key

Graph y = 1/(x-4). Identify the vertical asymptote, horizontal asymptote, intercepts, and sketch the branches.

Answer: Vertical asymptote: x = 4. Horizontal asymptote: y = 0. y-intercept: (0, -1/4). No x-intercept. Test points: x = 5 gives y = 1; x = 3 gives y = -1. The branch for x > 4 lies above y = 0 and approaches positive infinity as x approaches 4 from the right. The branch for x < 4 lies below y = 0 and approaches negative infinity as x approaches 4 from the left. Both branches approach y = 0 as x goes to positive or negative infinity.

Graph Rational Functions: Vertical and Horizontal Asymptotes

Original tasks 1, 2 / complete keys

Task 1

Graph y = 2/(x-1). Find the vertical asymptote, horizontal asymptote, intercepts, and describe the branches.

Answer: Vertical asymptote: x = 1. Horizontal asymptote: y = 0. y-intercept: (0, -2). No x-intercept. For x > 1, y is positive and approaches positive infinity as x approaches 1 from the right. For x < 1, y is negative and approaches negative infinity as x approaches 1 from the left. Both branches approach y = 0 at the far ends.

Reason: The denominator x-1 is zero at x = 1, giving the vertical asymptote. The constant term is 0, so the horizontal asymptote is y = 0. Substituting x = 0 gives y = 2/(0-1) = -2. Setting y = 0 gives 2/(x-1) = 0, which has no solution. Test x = 2 gives y = 2, and x = 0 gives y = -2, confirming branch positions.

Check: Verify that x = 1 makes the denominator zero, that y approaches 0 as x grows large, and that the plotted points (0, -2) and (2, 2) lie on the correct branches.

Task 2

Graph y = -1/(x+2). Find the vertical asymptote, horizontal asymptote, intercepts, and describe the branches.

Answer: Vertical asymptote: x = -2. Horizontal asymptote: y = 0. y-intercept: (0, -1/2). No x-intercept. For x > -2, the denominator is positive and the numerator is negative, so y is negative and approaches negative infinity as x approaches -2 from the right. For x < -2, the denominator is negative and the numerator is negative, so y is positive and approaches positive infinity as x approaches -2 from the left. Both branches approach y = 0 at the far ends.

Reason: The denominator x+2 is zero at x = -2. The constant term is 0, so the horizontal asymptote is y = 0. Substituting x = 0 gives y = -1/(0+2) = -1/2. Setting y = 0 gives -1/(x+2) = 0, which has no solution. Test x = -1 gives y = -1, and x = -3 gives y = 1, confirming branch positions.

Check: Check that x = -2 makes the denominator zero, that y approaches 0 as x grows large, and that the test points (-1, -1) and (-3, 1) match the computed values.

Graph Rational Functions: Vertical and Horizontal Asymptotes

Original tasks 3, 4 / complete keys

Task 3

Graph y = 1/(x-3) + 2. Find the vertical asymptote, horizontal asymptote, intercepts, and describe the branches.

Answer: Vertical asymptote: x = 3. Horizontal asymptote: y = 2. y-intercept: (0, 5/3). x-intercept: set y = 0, so 1/(x-3) = -2, giving x - 3 = -1/2, so x = 2.5. For x > 3, y is above 2 and approaches positive infinity as x approaches 3 from the right. For x < 3, y is below 2 and approaches negative infinity as x approaches 3 from the left. Both branches approach y = 2 at the far ends.

Reason: The denominator x-3 is zero at x = 3. The constant term is 2, so the horizontal asymptote is y = 2. Substituting x = 0 gives y = 1/(0-3) + 2 = -1/3 + 2 = 5/3. Setting y = 0 gives 1/(x-3) = -2, so x - 3 = -1/2 and x = 2.5. Test x = 4 gives y = 3, and x = 2 gives y = 1, confirming branch positions relative to y = 2.

Check: Verify the vertical asymptote at x = 3, the horizontal asymptote at y = 2, the intercepts (0, 5/3) and (2.5, 0), and that test points (4, 3) and (2, 1) lie on the correct branches.

Task 4

Graph y = -2/(x-1) - 1. Find the vertical asymptote, horizontal asymptote, intercepts, and describe the branches.

Answer: Vertical asymptote: x = 1. Horizontal asymptote: y = -1. y-intercept: (0, 1). x-intercept: set y = 0, so -2/(x-1) = 1, giving x - 1 = -2, so x = -1. For x > 1, the denominator is positive and the numerator is negative, so y is below -1 and approaches negative infinity as x approaches 1 from the right. For x < 1, the denominator is negative and the numerator is negative, so y is above -1 and approaches positive infinity as x approaches 1 from the left. Both branches approach y = -1 at the far ends.

Reason: The denominator x-1 is zero at x = 1. The constant term is -1, so the horizontal asymptote is y = -1. Substituting x = 0 gives y = -2/(0-1) - 1 = 2 - 1 = 1. Setting y = 0 gives -2/(x-1) = 1, so x - 1 = -2 and x = -1. Test x = 2 gives y = -3, and x = 0 gives y = 1, confirming branch positions relative to y = -1.

Check: Check the vertical asymptote at x = 1, the horizontal asymptote at y = -1, the intercepts (0, 1) and (-1, 0), and that test points (2, -3) and (0, 1) lie on the correct branches.

Graph Rational Functions: Vertical and Horizontal Asymptotes

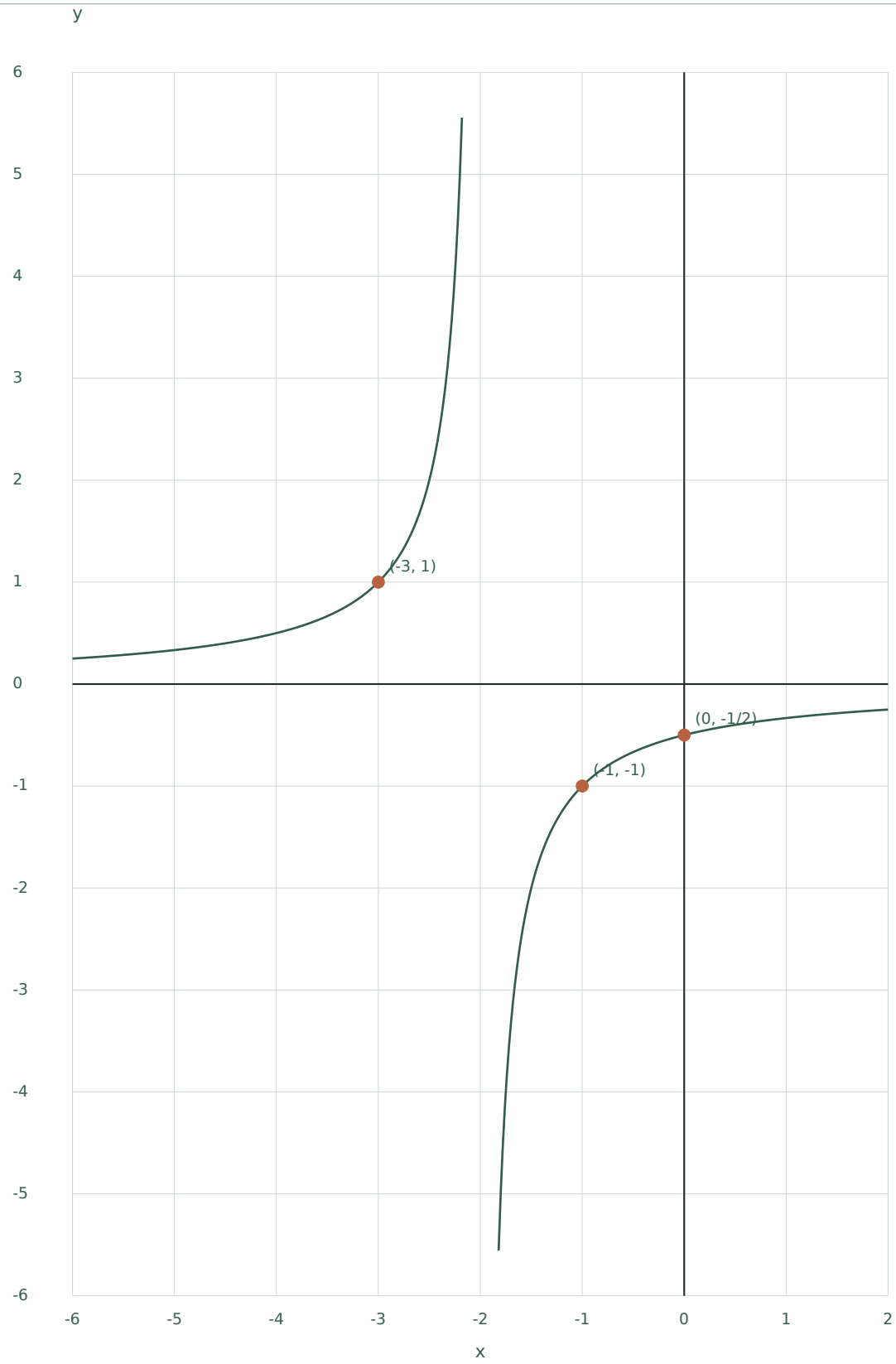
Task 1 / Answer Key Exercise 1: y = 2/(x-1)



Vertical asymptote x = 1; horizontal asymptote y = 0; no x-intercept.

Graph Rational Functions: Vertical and Horizontal Asymptotes

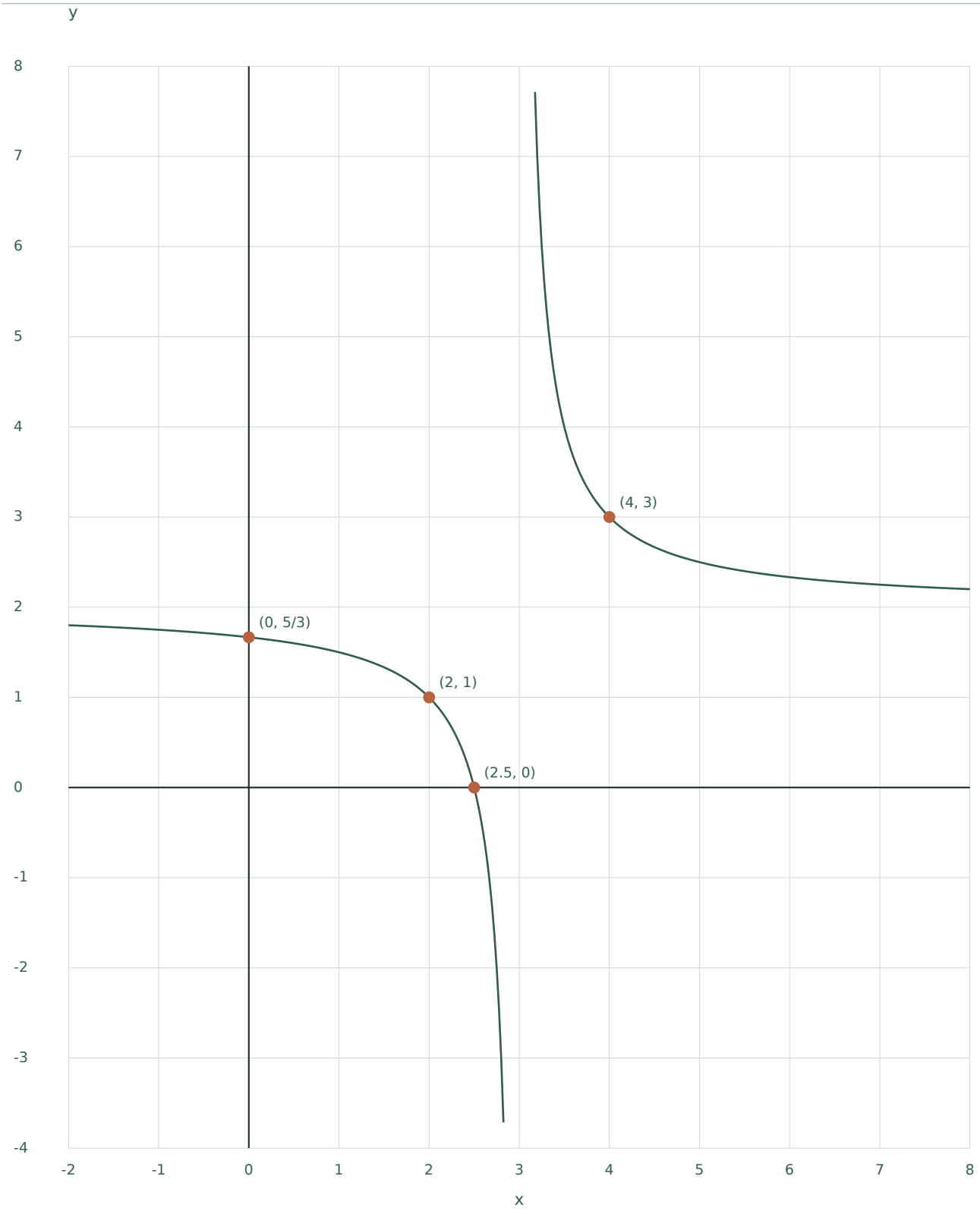
Task 2 / Answer Key Exercise 2: y = -1/(x+2)



Vertical asymptote x = -2; horizontal asymptote y = 0; no x-intercept.

Graph Rational Functions: Vertical and Horizontal Asymptotes

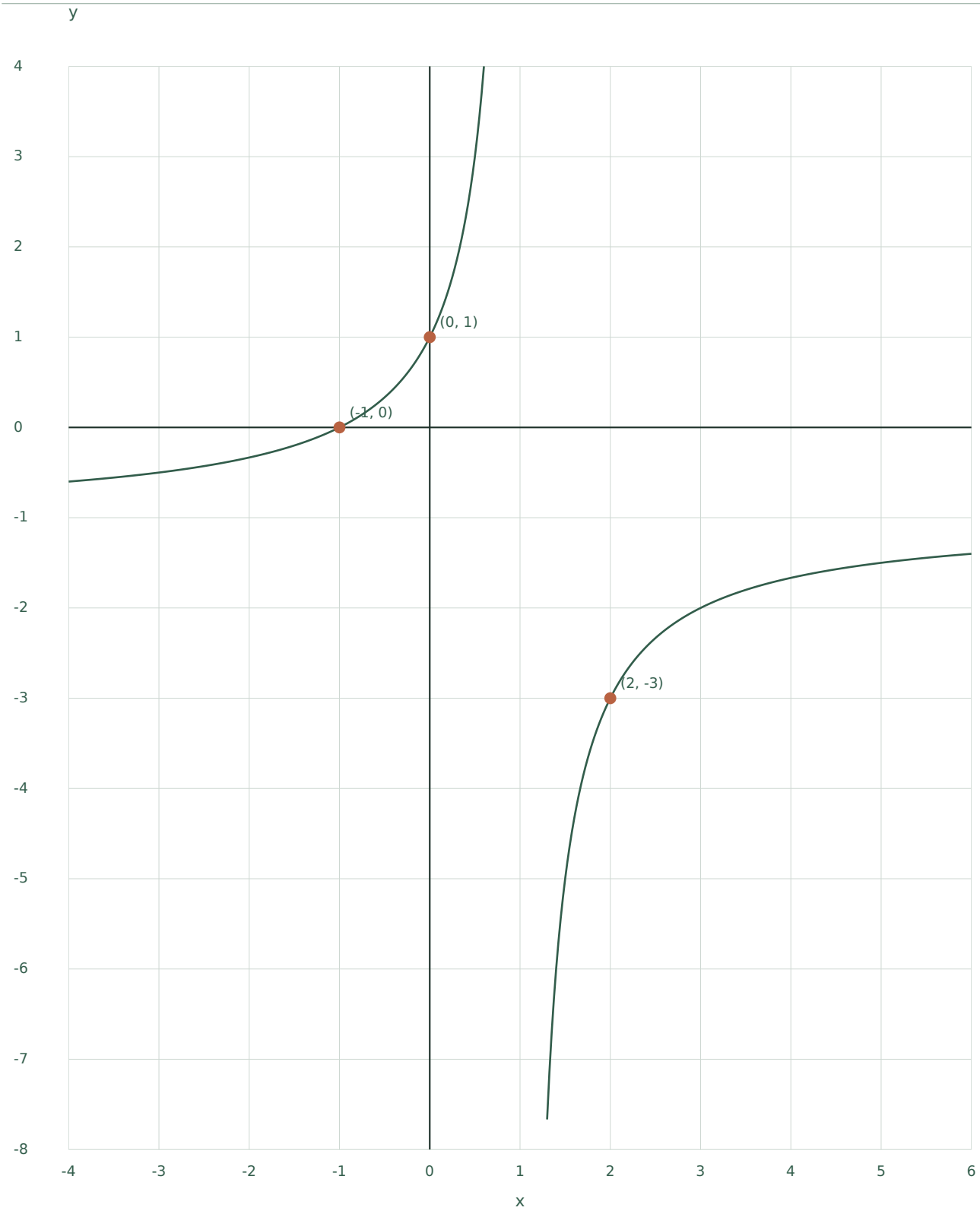
Task 3 / Answer Key Exercise 3: y = 1/(x-3) + 2



Vertical asymptote x = 3; horizontal asymptote y = 2.

Graph Rational Functions: Vertical and Horizontal Asymptotes

Task 4 / Answer Key Exercise 4: y = -2/(x-1) - 1



Vertical asymptote x = 1; horizontal asymptote y = -1.

Graph Rational Functions: Vertical and Horizontal Asymptotes

Task 1, Task 2, Task 3, Task 4 / Answer Key: Asymptotes, Intercepts, and Branch Descriptions

Text key for the four exercises and rubric criteria.

Exercise 1: y = 2/(x-1). Vertical asymptote x = 1; horizontal asymptote y = 0; y-intercept (0, -2); no x-intercept. For x > 1, y is positive and approaches positive infinity as x approaches 1 from the right. For x < 1, y is negative and approaches negative infinity as x approaches 1 from the left. Both branches approach y = 0 at the far ends.

Exercise 2: y = -1/(x+2). Vertical asymptote x = -2; horizontal asymptote y = 0; y-intercept (0, -1/2); no x-intercept. For x > -2, y is negative and approaches negative infinity as x approaches -2 from the right. For x < -2, y is positive and approaches positive infinity as x approaches -2 from the left. Both branches approach y = 0 at the far ends.

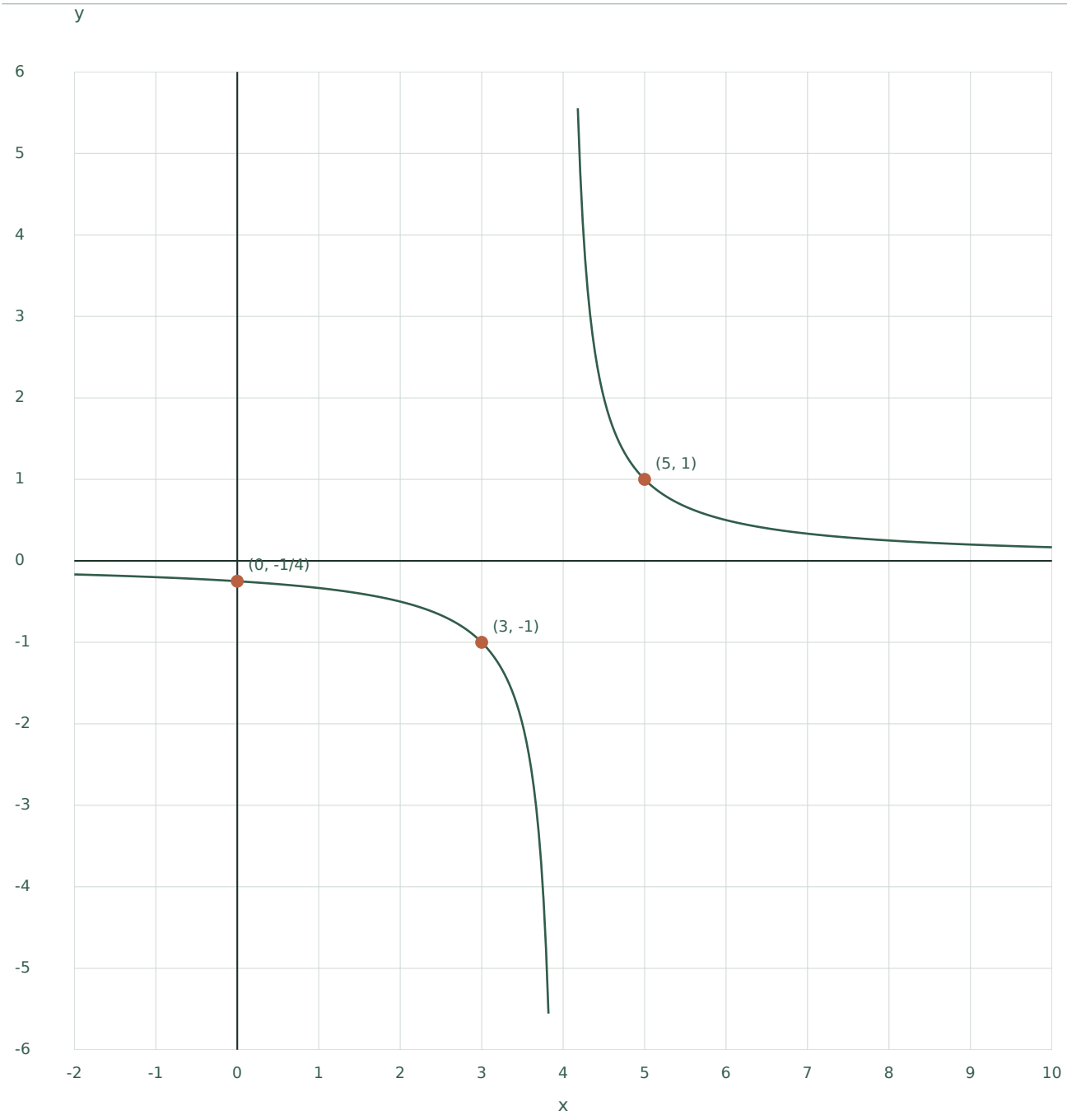
Exercise 3: y = 1/(x-3) + 2. Vertical asymptote x = 3; horizontal asymptote y = 2; y-intercept (0, 5/3); x-intercept (2.5, 0). For x > 3, y is above 2 and approaches positive infinity as x approaches 3 from the right. For x < 3, y is below 2 and approaches negative infinity as x approaches 3 from the left. Both branches approach y = 2 at the far ends.

Exercise 4: y = -2/(x-1) - 1. Vertical asymptote x = 1; horizontal asymptote y = -1; y-intercept (0, 1); x-intercept (-1, 0). For x > 1, y is below -1 and approaches negative infinity as x approaches 1 from the right. For x < 1, y is above -1 and approaches positive infinity as x approaches 1 from the left. Both branches approach y = -1 at the far ends.

Rubric: (1) Vertical asymptote correctly identified and drawn as a dashed line. (2) Horizontal asymptote correctly identified and drawn as a dashed line. (3) Intercepts computed accurately and plotted at correct coordinates. (4) Branches placed on correct sides of the vertical asymptote with correct sign relative to the horizontal asymptote. (5) End behavior near asymptotes matches the function's values. (6) Graph does not cross the vertical asymptote. (7) Test points used to

Graph Rational Functions: Vertical and Horizontal Asymptotes

Worked example / Worked Example: y = 1/(x-4)



Dashed asymptotes x = 4 and y = 0 are shown as guide lines; branches approach but do not cross x = 4.

Graph Rational Functions: Vertical and Horizontal Asymptotes

Source rubric / reflection

- [ ] Vertical asymptote correctly identified and drawn as a dashed line.
- [ ] Horizontal asymptote correctly identified and drawn as a dashed line.
- [ ] Intercepts computed accurately and plotted at correct coordinates.
- [ ] Branches placed on correct sides of the vertical asymptote with correct sign relative to the horizontal asymptote.
- [ ] End behavior near asymptotes matches the function's values.
- [ ] Graph does not cross the vertical asymptote.
- [ ] Test points used to verify branch positions.

Reflection: What did you check, what did you change, and what will you try next?