Sketching Derivative Graphs from Function Curves: A Visual Guide

Complete worked example / separate answer key

A function curve on a grid rises steeply from x=0 to x=2, reaches a flat top at x=2, then falls gently from x=2 to x=4. Sketch the derivative graph using a vertical scale where one grid square equals one unit of slope. Describe the sketch.

Answer: The derivative graph is above the axis from x=0 to x=2, with a high positive value near x=1 (about two squares up as a hypothetical example). At x=2 it crosses the axis (zero slope). From x=2 to x=4 it is below the axis, with a smaller negative value (about one square down) because the fall is gentler than the rise. The sketch is a smooth curve that starts positive, decreases to zero at x=2, and continues negative with a shallower depth.

Reason: The function is increasing before x=2, so the derivative is positive; steeper rise means larger positive height. At x=2 the tangent is horizontal, so the derivative is zero. After x=2 the function decreases, so the derivative is negative; gentler fall means smaller magnitude. The sign change from positive to negative at x=2 matches a local maximum.

Check: Count horizontal tangents: one at x=2, matching one x-intercept. Check intervals: increasing gives above-axis, decreasing gives below-axis. Compare steepness: rise steeper than fall, so positive height greater in magnitude than negative height.

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Original tasks 1, 2 / complete keys

Task 1

A function curve is the straight line through the points (0,0) and (2,4) on a grid where one square equals one unit horizontally and one unit vertically. Sketch its derivative graph on a grid with the same horizontal scale, using a vertical scale where one square equals one unit of slope. State the height of your derivative line as a hypothetical example.

Answer: The derivative graph is a horizontal line above the axis at a constant positive height. Since the line rises 4 units over 2 horizontal units, the slope is $4 \div 2 = 2$, so the derivative line sits two squares above the axis (hypothetical example using the stated scale).

Reason: A straight line has constant slope, so the derivative is constant. The slope is $(4 - 0) \div (2 - 0) = 2$, which is positive, so the derivative line is horizontal and above the axis at height 2 on the chosen scale.

Check: Verify the derivative is horizontal and above the axis. Recompute the slope as rise ÷ run = $4 \div 2 = 2$ and confirm the line is drawn two squares up. Confirm the height is labeled and consistent with your chosen scale.

Task 2

A function curve is a parabola opening upward with its vertex at (1,0). It passes through (0,1) and (2,1), falling for $x<1$ and rising for $x>1$. Sketch the derivative graph on a grid where one square equals one unit horizontally and one unit vertically, and describe its shape and zero location.

Answer: The derivative graph is a straight line with positive slope, crossing the axis at $x=1$. It is below the axis for $x<1$ and above for $x>1$. Using a labeled vertical scale where one square equals one unit of slope (hypothetical example), the derivative is about -2 at $x=0$ and about $+2$ at $x=2$, so the derivative sketch is a line through (1,0) rising 4 slope units over 2 horizontal units, from -2 at $x=0$ to $+2$ at $x=2$.

Reason: The parabola has a horizontal tangent at its vertex $x=1$, so the derivative is zero there. The function decreases before $x=1$ (negative slope) and increases after (positive slope). For a parabola, the slope changes at a constant rate, so the derivative is linear.

Check: Check one x-intercept at $x=1$. Confirm the derivative is negative left of $x=1$ and positive right of it. Confirm the derivative line has positive slope and passes through (1,0). Verify that any heights you drew are labeled as approximate and consistent with your chosen scale.

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Original tasks 3, 4 / complete keys

Task 3

A function curve is made of three straight segments meeting at sharp corners: it rises from (0,0) to (1,1), then rises more steeply from (1,1) to (2,4), then is flat from (2,4) to (3,4). Sketch the derivative graph on a grid where one square equals one unit horizontally and one unit vertically, using a labeled vertical scale, and describe the heights on each interval.

Answer: On (0,1) the slope is $(1 - 0) \div (1 - 0) = 1$, so the derivative is positive and one square up. On (1,2) the slope is $(4 - 1) \div (2 - 1) = 3$, so the derivative is positive and three squares up. On (2,3) the slope is 0, so the derivative is zero. Because the segments meet at sharp corners at x=1 and x=2, the derivative is undefined there; show a gap or vertical jump at each and label it undefined. The sketch otherwise shows a step up in positive height, then a drop to zero.

Reason: Increasing intervals give positive derivative values. The steeper segment has the larger slope, so its derivative height is greater. The flat segment has zero slope, so the derivative is zero there. Sharp corners at x=1 and x=2 mean the derivative is undefined at those x-values.

Check: Confirm the derivative is above the axis on (0,2) and zero on (2,3). Check that the height on (1,2) is greater than on (0,1) by comparing slopes 3 and 1. Count one flat section matching one zero interval. Verify that x=1 and x=2 are marked as undefined (gap or vertical jump) because the segments meet at sharp corners there.

Task 4

A function curve has a local maximum at x=2 and a local minimum at x=5. It rises before x=2, falls between x=2 and x=5, and rises after x=5. On a grid where one square equals one unit horizontally, sketch the derivative graph using a labeled vertical scale, and mark its zeros and sign changes.

Answer: The derivative crosses the axis at x=2 from positive to negative, and at x=5 from negative to positive. It is above the axis before x=2, below between x=2 and x=5, and above after x=5. Heights are approximate; for example, the positive height before x=2 may be about two squares up and the negative dip between x=2 and x=5 about one square down (hypothetical example).

Reason: At a local maximum the slope changes from positive to negative, so the derivative changes from positive to negative. At a local minimum the slope changes from negative to positive. These sign changes determine the derivative graph's position relative to the axis.

Check: Verify two x-intercepts at x=2 and x=5. Check the sign pattern: positive, negative, positive. Confirm the sign changes match the maximum and minimum, and that your vertical scale is labeled and used consistently.

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Task 1, Task 2, Task 3, Task 4 / Answer Key: Derivative Sketches

Sample derivative sketches and reasoning; sketches approximate.

Exercise 1: derivative is a horizontal line above the axis at height 2 (hypothetical example). Slope = 4 ÷ 2 = 2.

Exercise 2: derivative is a line through (1,0) with positive slope; about −2 at x=0 and +2 at x=2 (hypothetical example).

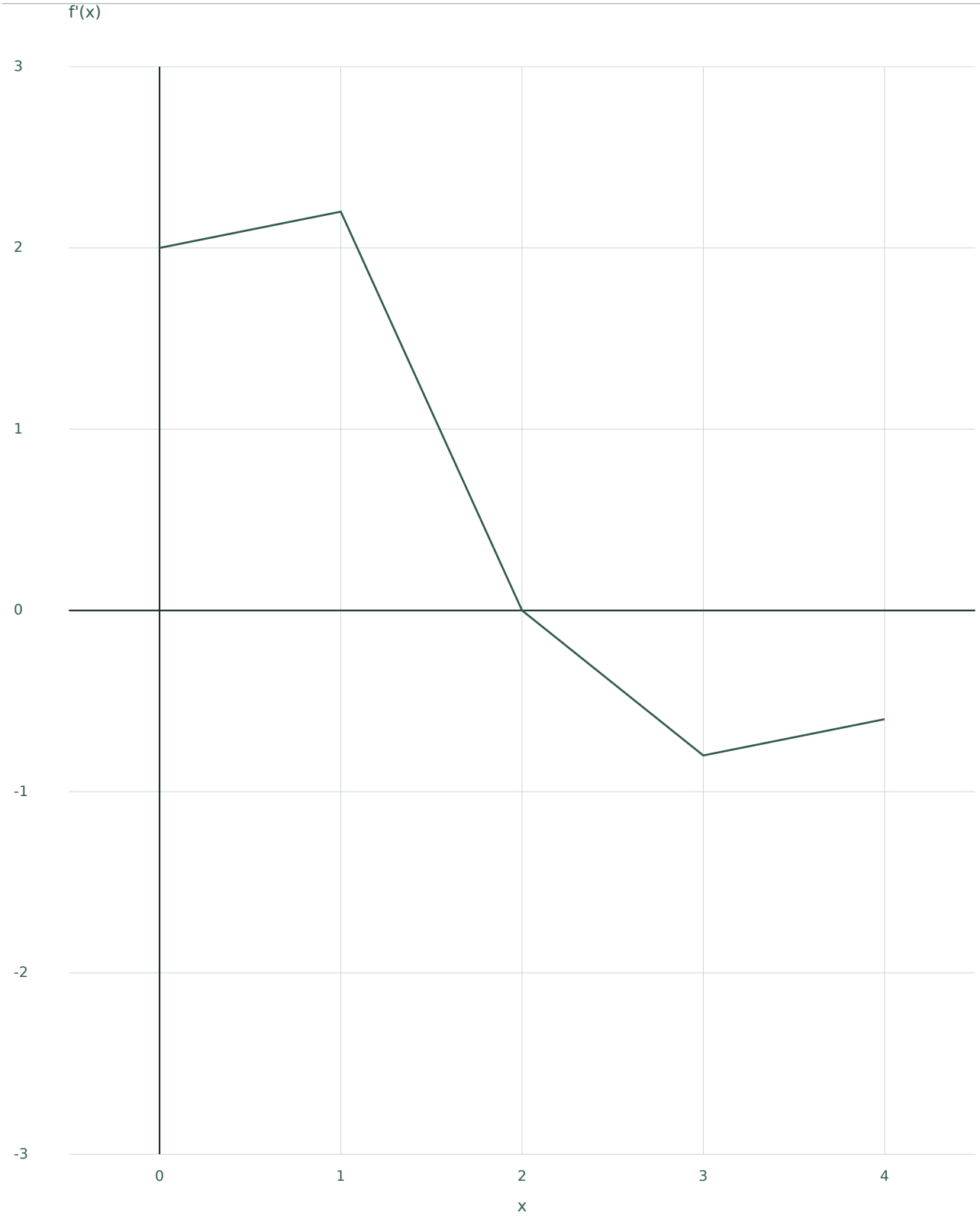
Exercise 3: derivative is 1 on (0,1), 3 on (1,2), 0 on (2,3); undefined at x=1 and x=2 (gap or vertical jump).

Exercise 4: derivative crosses axis at x=2 from positive to negative and at x=5 from negative to positive; sign pattern +, −, +.

Rubric: zeros at horizontal tangents; above axis on increasing intervals, below on decreasing; relative steepness preserved with labeled consistent scale; sign changes match maxima/minima; smooth where function smooth; undefined at sharp corners; valid alternative scales accepted if labeled and consistent.

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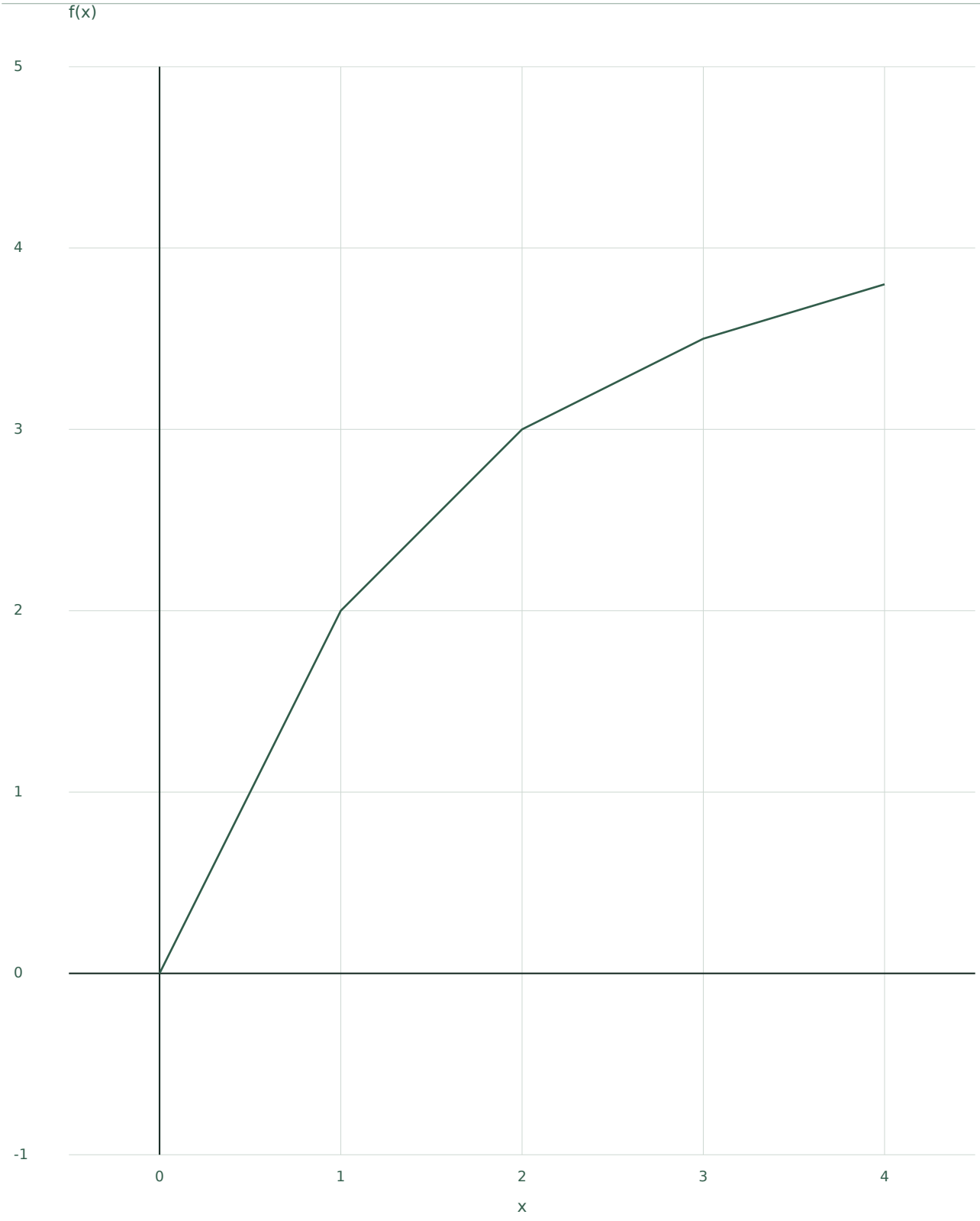
Worked example / Worked Example: Derivative Sketch



Vertical scale: one square = one unit of slope (hypothetical example).

Sketching Derivative Graphs from Function Curves: A Visual Guide

Worked example / Worked Example: Function Curve



Illustrative smooth curve approximated by polyline; flat top near x=2.

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Source rubric / reflection

- [ ] Derivative zeros are placed at x-values where the function curve has horizontal tangents.
- [ ] Derivative is above the axis on intervals where the function is increasing and below where it is decreasing.
- [ ] Relative steepness is preserved: steeper function slopes correspond to larger derivative magnitudes, using a labeled consistent scale.
- [ ] Sign changes at derivative zeros match local maxima (positive to negative) and local minima (negative to positive).
- [ ] Sketch is smooth where the function is smooth, and undefined points are marked where the function has sharp corners.
- [ ] Valid alternative scales or segment styles are accepted if labeled and internally consistent.

Reflection: What did you check, what did you change, and what will you try next?