How to Draw Contour Lines of a Multivariable Function

Original tasks 1, 2 / complete keys

Task 1

Draw contour lines for f(x,y)=x+y at c=-2,0,2 on a grid from -3 to 3. Label each line.

Answer: The levels are parallel lines y=-2-x, y=-x, and y=2-x. Sample points: for c=-2, (-2,0),(0,-2),(1,-3); for c=0, (-1,1),(0,0),(1,-1); for c=2, (0,2),(1,1),(2,0). Connect each set with a straight line and label c=-2, c=0, c=2.

Reason: Each level equation is linear, so the contour is a straight line. Plotting three points per line fixes its position and slope.

Check: Substitute one point per line into x+y and confirm it equals the labeled c. Check that lines are parallel and do not cross.

Task 2

Draw contour lines for f(x,y)=xy at c=1 and c=-1 on a grid from -3 to 3, excluding x=0 and y=0. Label each branch.

Answer: For c=1, plot (0.5,2),(1,1),(2,0.5) and (-0.5,-2),(-1,-1),(-2,-0.5). For c=-1, plot (0.5,-2),(1,-1),(2,-0.5) and (-0.5,2),(-1,1),(-2,0.5). Connect points in each quadrant smoothly as hyperbola branches. Mark axes with dashed lines to show exclusion. Label c=1 on the first-quadrant branch and c=-1 on the second-quadrant branch, with matching labels on opposite branches.

Reason: The level xy=c gives y=c/x, a hyperbola. Positive c gives branches in quadrants I and III; negative c gives branches in quadrants II and IV.

Check: Pick one point per branch and verify the product equals the labeled c. Confirm branches do not cross the axes and are not connected across quadrants.

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Original tasks 3, 4 / complete keys

Task 3

Draw contour lines for f(x,y)=x^2+y^2 at c=0 and c=2 on a grid from -3 to 3. Explain what the c=0 contour is.

Answer: For c=2, the contour is a circle of radius sqrt(2) approximately 1.41. Plot (1.41,0),(0,1.41),(-1.41,0),(0,-1.41) and connect smoothly. For c=0, the only real point is (0,0), so draw a labeled dot at the origin and state that the level is a single point. Label the circle c=2 and the dot c=0.

Reason: x^2+y^2=c has real solutions only when c is at least 0. At c=0, both squares must be zero, giving the origin.

Check: Substitute the plotted circle points into x^2+y^2 and confirm they equal 2. Confirm the dot at the origin gives 0.

Task 4

Draw contour lines for f(x,y)=y-x^2 at c=0 and c=1 on a grid from -2 to 2 in x and 0 to 5 in y. Label each parabola.

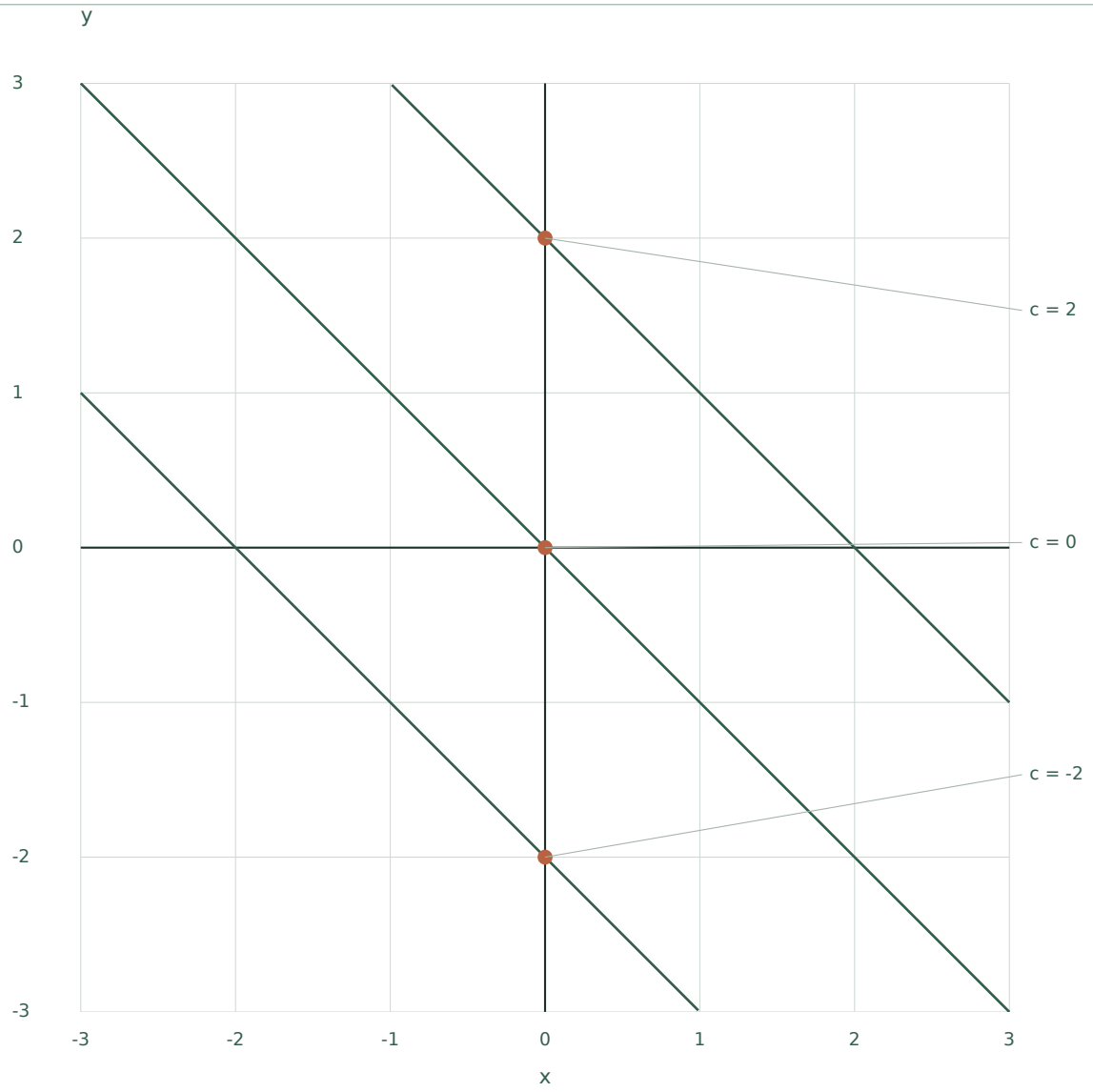
Answer: For c=0, y=x^2. Plot (-2,4),(-1,1),(0,0),(1,1),(2,4) and connect smoothly. For c=1, y=x^2+1. Plot (-2,5),(-1,2),(0,1),(1,2),(2,5) and connect smoothly. Label the lower parabola c=0 and the upper parabola c=1. State that both are parabolas opening upward.

Reason: Solving y-x^2=c gives y=x^2+c, a parabola shifted vertically by c. The two levels are parallel parabolas.

Check: Substitute one point per parabola into y-x^2 and confirm it equals the labeled c. Check that the c=1 parabola is one unit above the c=0 parabola at each x. Confirm all plotted points lie within the stated grid ranges.

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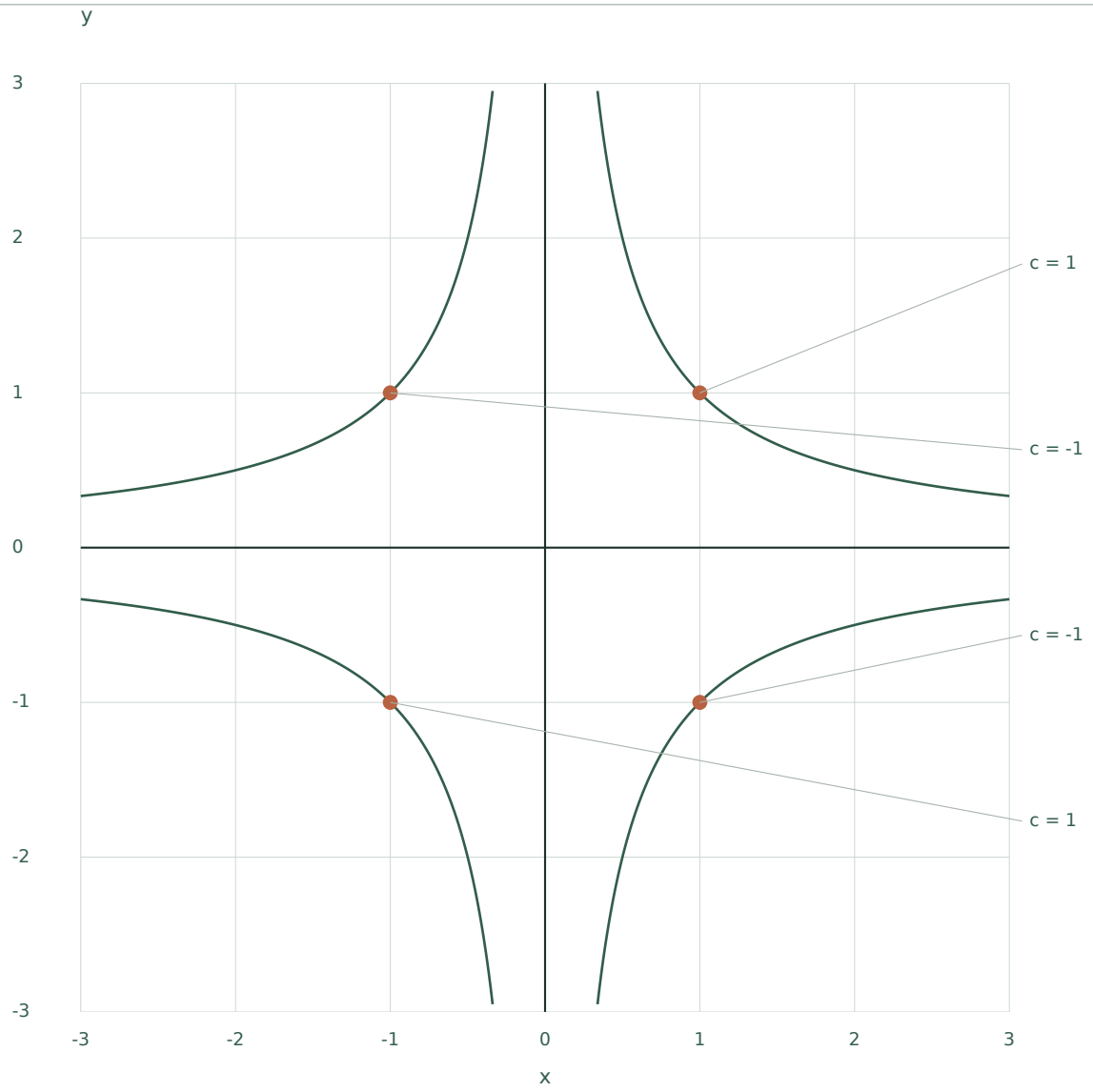
Task 1 / complete graph key



Each label identifies the contour through its marked point. Coordinate labels sit outside the plot; thin grey leaders are annotations, not function curves.

How to Draw Contour Lines of a Multivariable Function

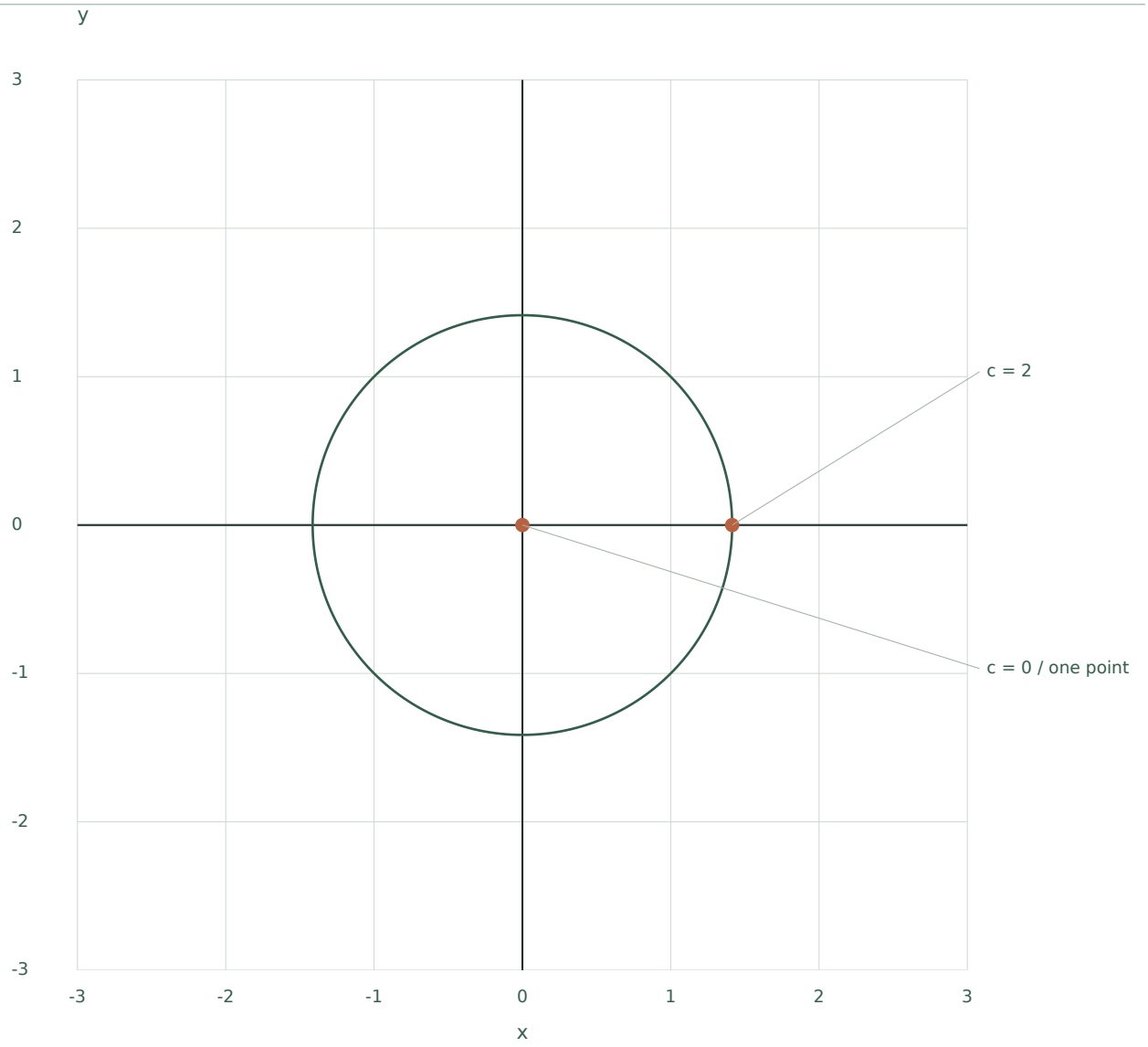
Task 2 / complete graph key



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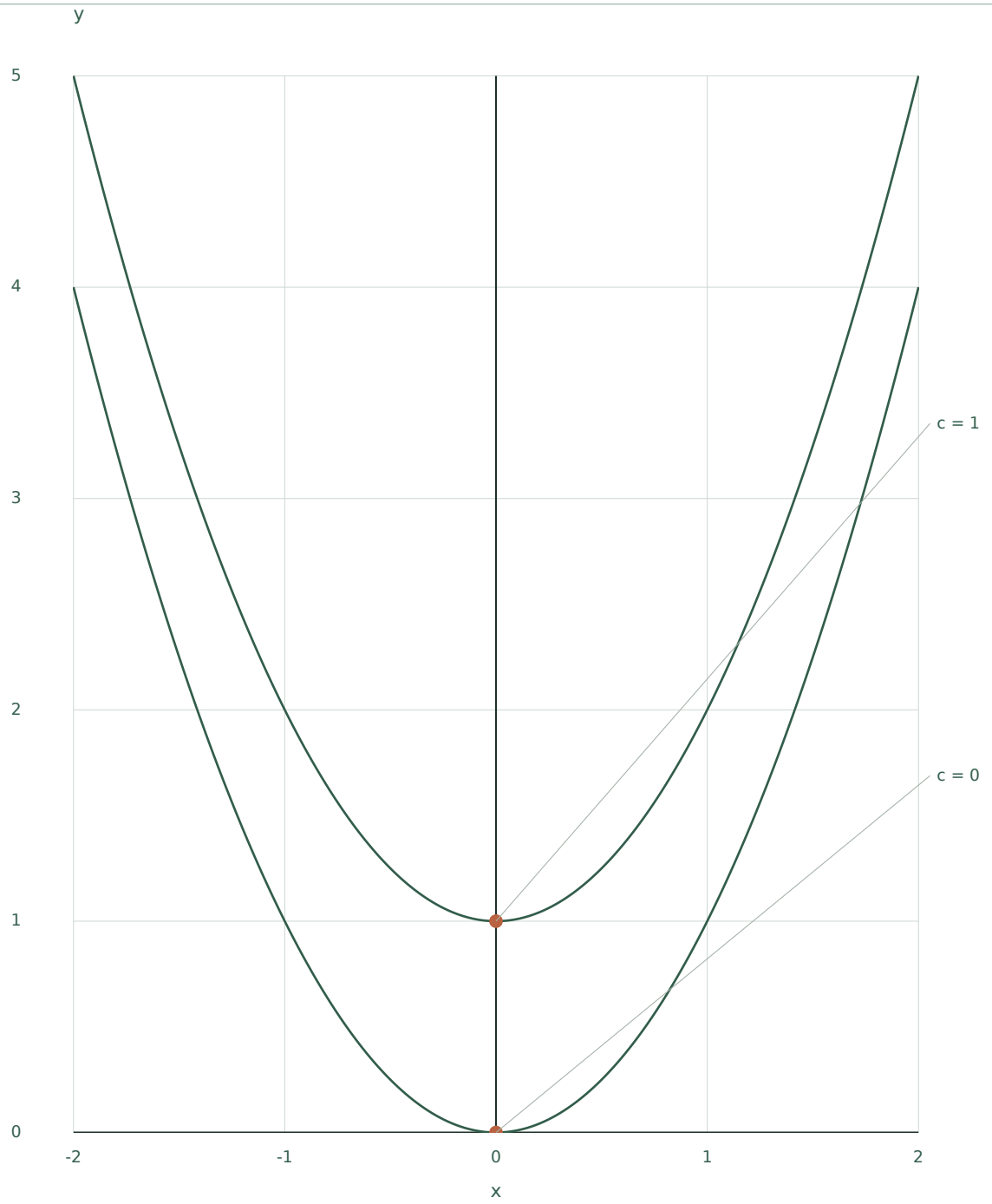
Task 3 / complete graph key



Each label identifies the contour through its marked point. For this radial function, c = 0 is a single origin point. Coordinate labels sit outside the plot; thin grey leaders are annotations, not function curves.

How to Draw Contour Lines of a Multivariable Function

Task 4 / complete graph key



Each label identifies the contour through its marked point. Coordinate labels sit outside the plot; thin grey leaders are annotations, not function curves.

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Source rubric / reflection

- [ ] Level equations are written correctly for each chosen c.
- [ ] At least four points per contour are plotted and satisfy f(x,y)=c.
- [ ] Curves are smooth and match the correct family (circle, line, hyperbola).
- [ ] Contours are labeled with their c values and axes have consistent units.
- [ ] A stated line convention is followed, and valid alternative curve shapes are accepted if points and labels are correct.

Reflection: What did you check, what did you change, and what will you try next?