Draw Contour Lines of a Quadratic Surface Model

Original worked example / unchanged complete key

Prompt: For f(x, y) = x^2 + 2y^2 on the grid x, y from -2 to 2 with step 0.5, draw the contour for level 4. List the exact and estimated points you would mark, and describe the shape.

Answer: Along the x-axis (y = 0), solve x^2 = 4, giving x = 2 and x = -2, both exact grid points. Along the y-axis (x = 0), solve 2y^2 = 4, giving y^2 = 2, so y ≈ 1.414 and y ≈ -1.414, estimated between 1.0 and 1.5 and between -1.5 and -1.0. Check interior grid points: (1.5, 1.0) gives 2.25 + 2 = 4.25, not 4; (1.0, 1.0) gives 1 + 2 = 3; (1.5, 0.5) gives 2.25 + 0.5 = 2.75. No interior grid intersection equals 4 exactly. Mark the four axis points and connect them with a smooth ellipse centered at the origin, wider along the x direction. Label the contour “f = 4” horizontally near the point (2, 0).

Reason: The level 4 contour is the set of points where x^2 + 2y^2 = 4. Solving along the axes gives the intercepts, and checking interior grid points confirms that no other exact marks are available. The ellipse is wider along x because the x^2 coefficient is smaller, so the contour extends farther in the x direction than in the y direction.

Check: Substitute (2, 0) into f: 4 + 0 = 4, correct. Substitute (0, 1.414) approximately: 0 + 2(2) = 4, correct within estimation. Confirm symmetry about both axes and that the contour does not cross the level 2 or level 8 contours.