Draw Contour Lines of a Quadratic Surface Model

Original tasks 1, 2 / complete keys

Task 1

For f(x, y) = x^2 + 2y^2 on the grid x, y from -2 to 2 with step 0.5, draw the contour for level 1. List the exact and estimated points you would mark, and describe the shape.

Answer: Along the x-axis, x^2 = 1 gives x = 1 and x = -1, both exact. Along the y-axis, 2y^2 = 1 gives y ≈ 0.707 and y ≈ -0.707, estimated between 0.5 and 1.0 and between -1.0 and -0.5. No interior grid intersection equals 1 exactly: (0.5, 0.5) gives 0.25 + 0.5 = 0.75; (1.0, 0.5) gives 1 + 0.5 = 1.5. Mark the four axis points and connect them with a small ellipse centered at the origin, wider along x. Label “f = 1” horizontally near (1, 0).

Reason: The level 1 contour satisfies x^2 + 2y^2 = 1. Axis intercepts give the marks, and interior checks confirm no exact grid intersections. The ellipse is small and stretched along x because the x^2 coefficient is smaller.

Check: Substitute (1, 0): 1 + 0 = 1, correct. Substitute (0, 0.707) approximately: 0 + 2(0.5) = 1, correct within estimation. Confirm symmetry and that the contour lies inside the level 2 contour.

Task 2

For f(x, y) = x^2 + 2y^2 on the grid x, y from -2 to 2 with step 0.5, draw the contour for level 2. List the exact and estimated points you would mark, and describe the shape.

Answer: Along the x-axis, x^2 = 2 gives x ≈ 1.414 and x ≈ -1.414, estimated between 1.0 and 1.5 and between -1.5 and -1.0. Along the y-axis, 2y^2 = 2 gives y = 1 and y = -1, both exact. Check interior grid points: (1.0, 0.5) gives 1 + 0.5 = 1.5; (1.5, 0.5) gives 2.25 + 0.5 = 2.75; (1.0, 1.0) gives 1 + 2 = 3. No interior grid intersection equals 2 exactly. Mark the four axis points and connect them with an ellipse centered at the origin, wider along x. Label “f = 2” horizontally near (0, 1).

Reason: The level 2 contour satisfies x^2 + 2y^2 = 2. Axis intercepts give the marks, and interior checks confirm no exact grid intersections. The ellipse is larger than level 1 and still stretched along x.

Check: Substitute (0, 1): 0 + 2 = 2, correct. Substitute (1.414, 0) approximately: 2 + 0 = 2, correct within estimation. Confirm symmetry and that the contour lies between level 1 and level 4.

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Original tasks 3, 4 / complete keys

Task 3

For f(x, y) = x^2 + 2y^2 on the grid x, y from -2 to 2 with step 0.5, draw the contour for level 8. Describe which parts of the contour are visible on the grid and list the exact and estimated points you would mark.

Answer: Along the y-axis, 2y^2 = 8 gives y = 2 and y = -2, both exact grid points at the top and bottom edges. Along the x-axis, x^2 = 8 gives x ≈ 2.828 and x ≈ -2.828, which lie outside the grid, so those intercepts are not visible. Check interior grid points: (2, 1) gives 4 + 2 = 6; (2, 1.5) gives 4 + 4.5 = 8.5; (1.5, 1.5) gives 2.25 + 4.5 = 6.75. No interior grid intersection equals 8 exactly. Mark (0, 2) and (0, -2) and draw two arcs near the top and bottom edges, curving outward toward the left and right edges but not reaching them. Label “f = 8” horizontally near (0, 2).

Reason: The level 8 contour satisfies x^2 + 2y^2 = 8. The y-intercepts are exact grid points, but the x-intercepts lie outside the grid, so only two arcs are visible. Interior checks confirm no other exact marks.

Check: Substitute (0, 2): 0 + 8 = 8, correct. Substitute (2, 1.5): 4 + 4.5 = 8.5, not 8, so that point is not on the contour. Confirm the arcs are symmetric about both axes and lie outside the level 4 contour.

Task 4

For f(x, y) = x^2 + 2y^2 on the grid x, y from -2 to 2 with step 0.5, explain why the level 0 contour is a single point and describe how you would mark it. Then state one check that confirms your marking is correct.

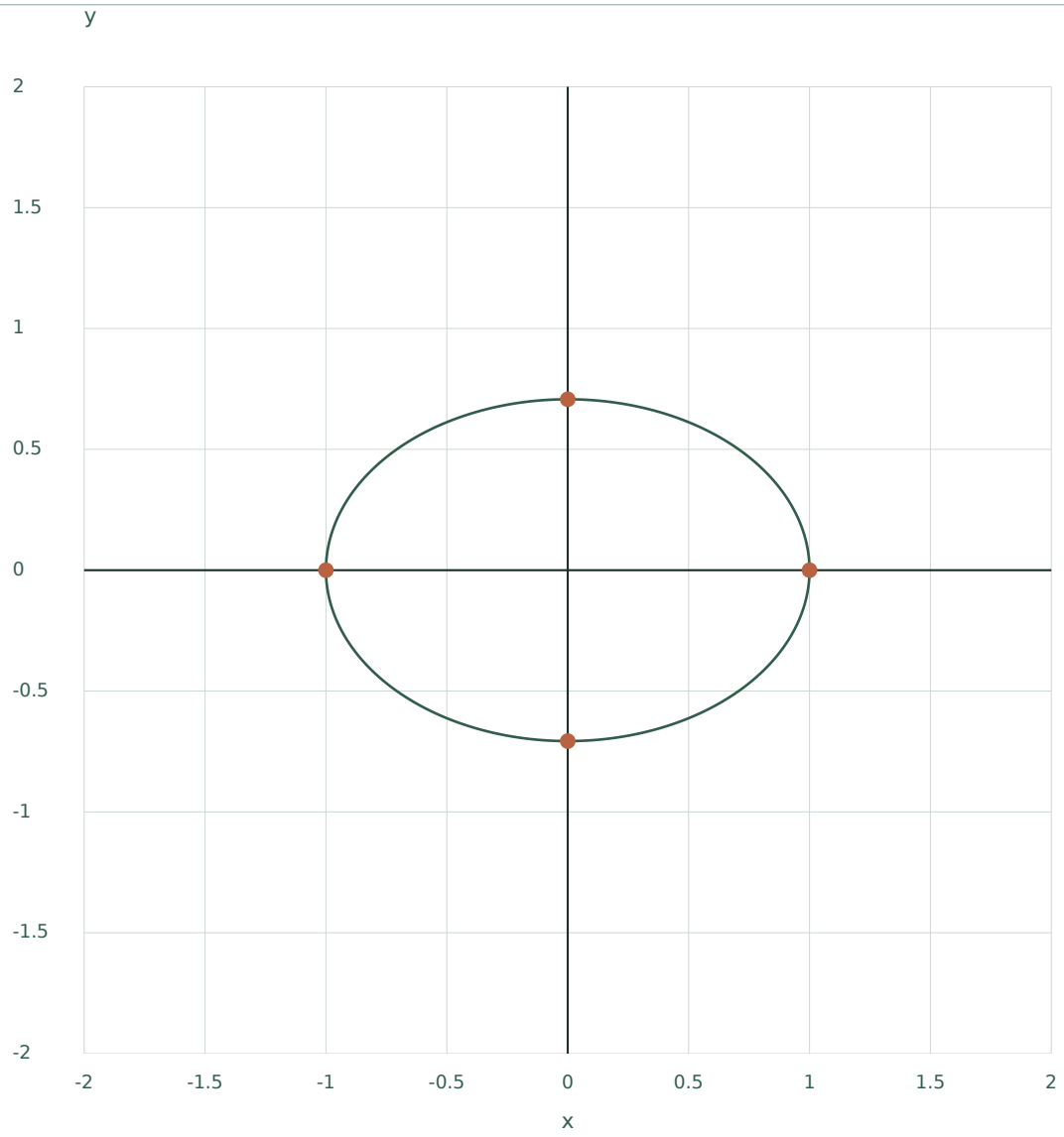
Answer: The level 0 contour satisfies x^2 + 2y^2 = 0. Since both terms are nonnegative, the only solution is x = 0 and y = 0. Mark the origin with a dot and label it “f = 0” horizontally nearby. Do not draw a curve. A check is to substitute (0, 0) into the function: 0 + 0 = 0, correct. Also confirm that no other grid intersection gives 0, since any nonzero x or y makes at least one term positive.

Reason: The sum of two nonnegative terms is zero only when both terms are zero. The origin is the only point on the level 0 contour, so it is a degenerate contour, not a curve.

Check: Substitute (0, 0) into f and confirm the result is 0. Check a nearby grid point such as (0.5, 0) gives 0.25, not 0, confirming the contour is a single point.

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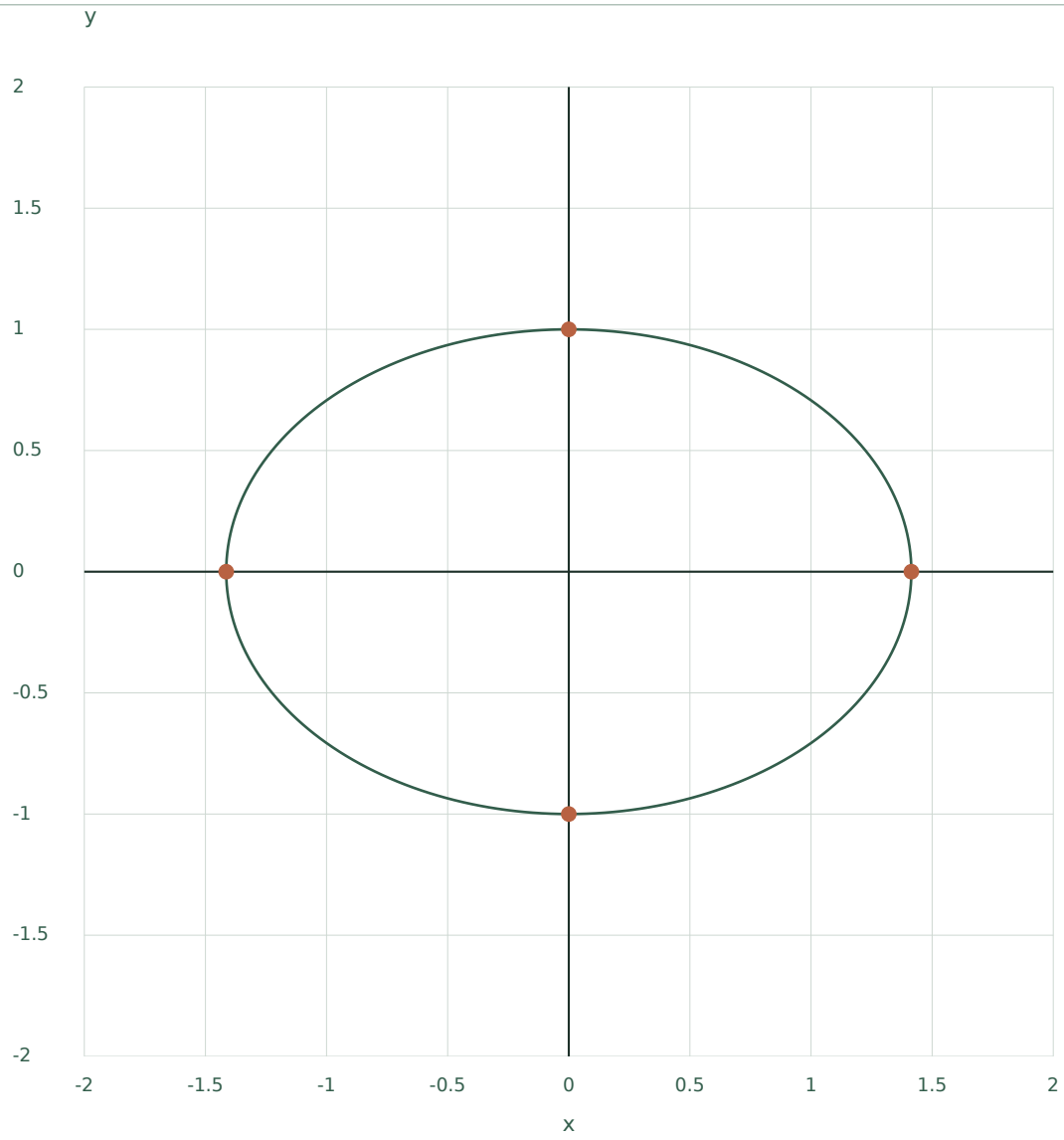
Level 1 contour of f(x,y)=x^2+2y^2



Ellipse centered at origin, wider along x; axis intercepts marked.

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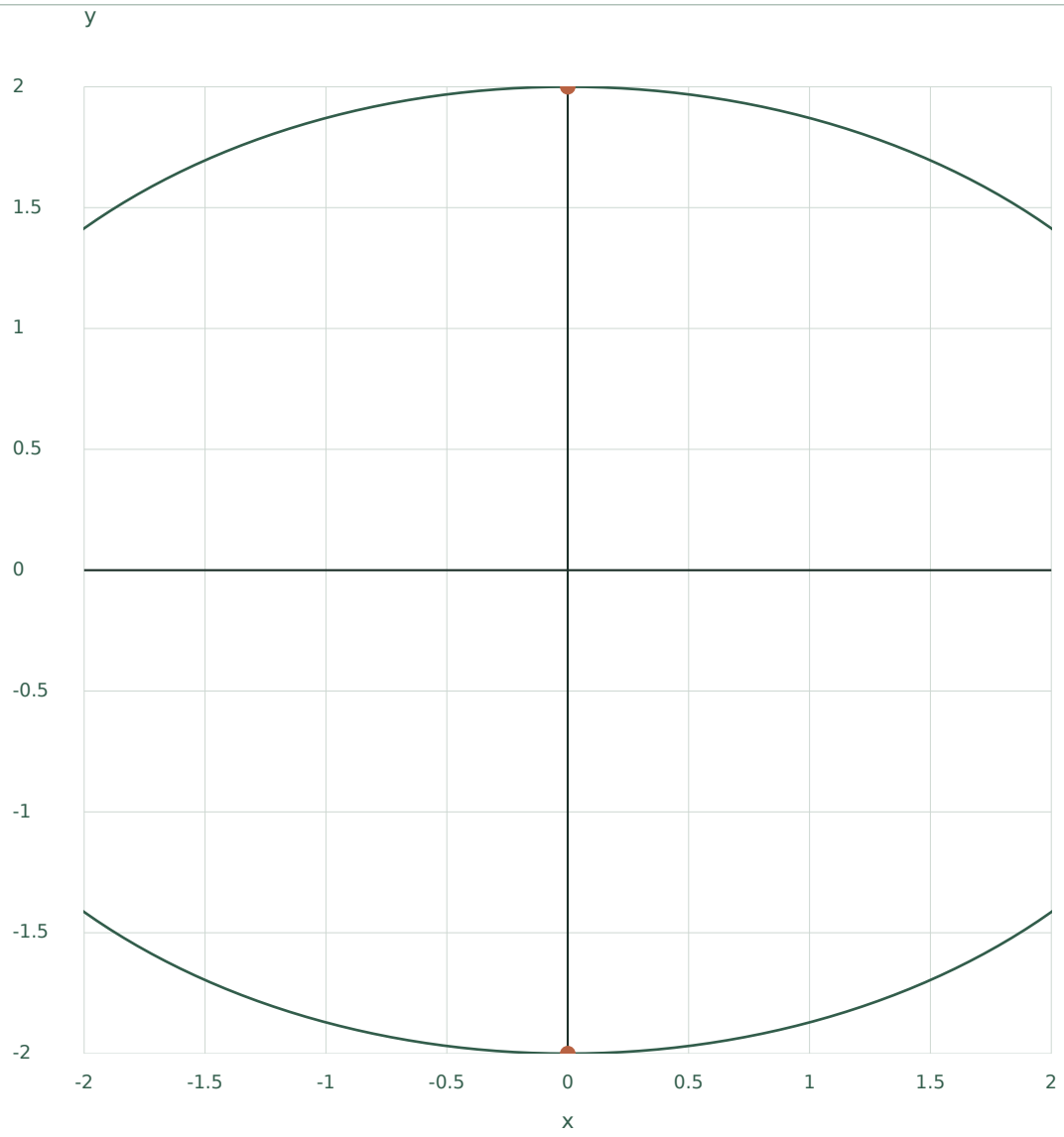
Level 2 contour of f(x,y)=x^2+2y^2



Ellipse centered at origin, wider along x; axis intercepts marked.

Draw Contour Lines of a Quadratic Surface Model

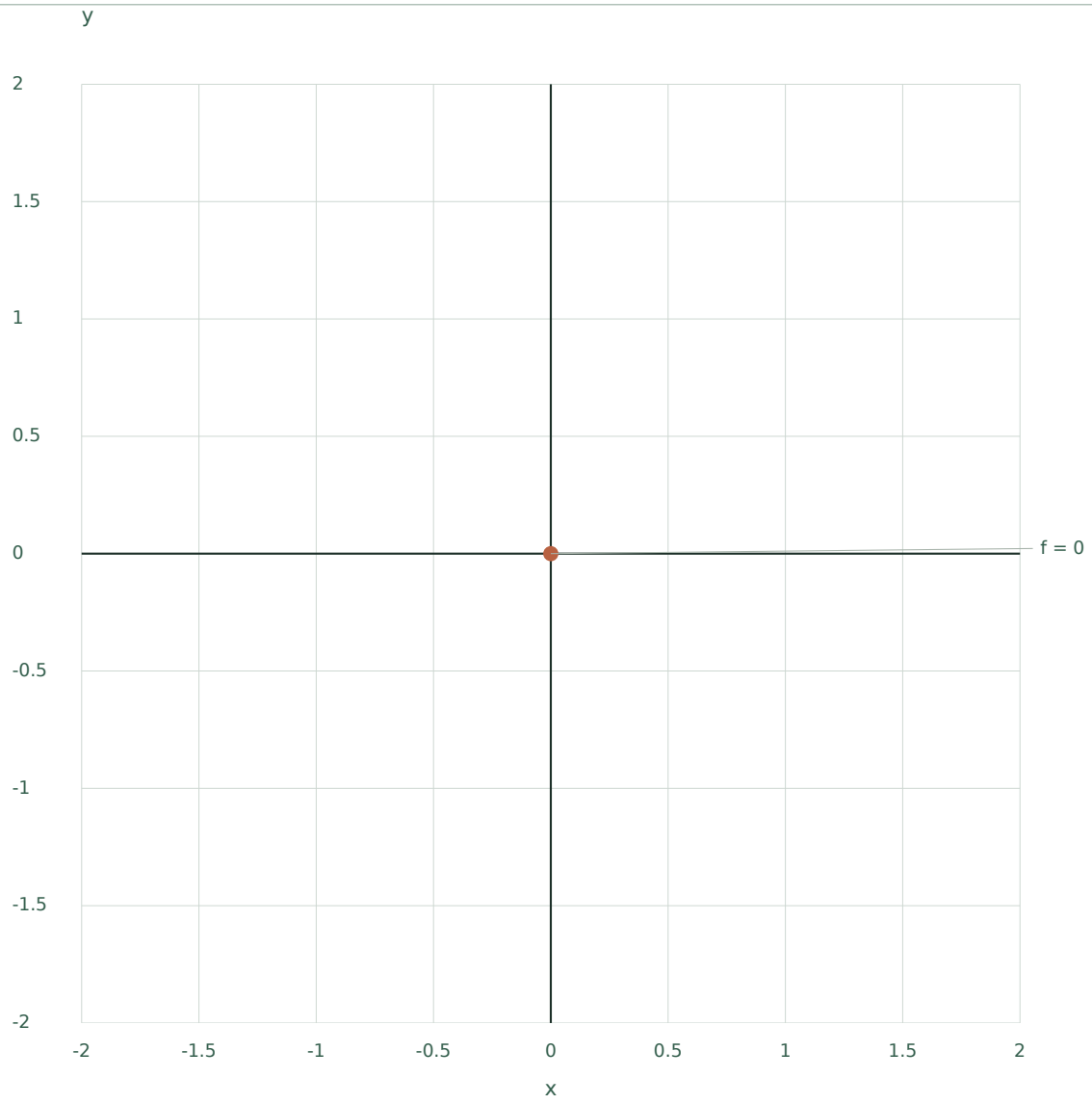
Visible arcs of level 8 contour of f(x,y)=x^2+2y^2



Only top and bottom arcs visible within grid; x-intercepts lie outside.

Draw Contour Lines of a Quadratic Surface Model

Level 0 contour: single point at origin



Degenerate contour: single point at origin, no curve. Coordinate labels sit outside the plot; thin grey leaders are annotations, not function curves.

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Source rubric / reflection

- [ ] Grid and axes are labeled with consistent units and the function is written clearly.
- [ ] Each chosen level is evaluated at grid intersections and along axes, with exact and estimated points distinguished.
- [ ] Contours are smooth, do not cross, and pass through all marks of the same level.
- [ ] Each contour is labeled with its level value using horizontal text and a leader line if needed.
- [ ] The drawing shows correct symmetry about both axes and spacing that reflects the function’s rate of change.
- [ ] Handwriting and line style are legible; valid alternative letterforms and minor stylistic differences are accepted.

Reflection: What did you check, what did you change, and what will you try next?