Compare Cubic and Reciprocal Graph Shapes: Paired Plotting Worksheet

Original tasks 1, 2, 3 / complete keys

Task 1

Compare y = x^3 and y = 1/x on one grid using x-values -2, -1, 1, 2. State the asymptotes and whether each curve crosses the x-axis.

Answer: Cubic points: (-2,-8), (-1,-1), (1,1), (2,8). Reciprocal points: (-2,-0.5), (-1,-1), (1,1), (2,0.5). The reciprocal has asymptotes x = 0 and y = 0. The cubic crosses the x-axis at (0,0); the reciprocal never crosses the x-axis.

Reason: Substitute each x-value into each equation. The cubic is continuous and passes through the origin. The reciprocal is undefined at zero and has two branches approaching the axes.

Check: Verify each coordinate by substitution and confirm no reciprocal point has x = 0 or y = 0.

Task 2

Compare y = x^3 - 4x and y = 2/x using x-values -2, -1, 1, 2. Identify any turning points and asymptotes.

Answer: Cubic points: (-2,0), (-1,3), (1,-3), (2,0). Reciprocal points: (-2,-1), (-1,-2), (1,2), (2,1). The cubic has turning points near x ≈ -1.155 and x ≈ 1.155 (at x = ±2/√3), with approximate coordinates (-1.155, 3.079) and (1.155, -3.079). The reciprocal has asymptotes x = 0 and y = 0.

Reason: Compute y = x^3 - 4x and y = 2/x for each listed x. The cubic changes direction near x ≈ ±1.155, which lies between the plotted points -2 and -1 and between 1 and 2; the reciprocal splits into two branches.

Check: Check that cubic values satisfy y = x^3 - 4x and reciprocal values satisfy xy = 2; verify the turning points by confirming the sign of the slope changes around x ≈ ±1.155, and confirm asymptote labels.

Task 3

Compare y = x^3 + x and y = -1/x using x-values -2, -1, 1, 2. Describe end behavior and asymptote behavior.

Answer: Cubic points: (-2,-10), (-1,-2), (1,2), (2,10). Reciprocal points: (-2,0.5), (-1,1), (1,-1), (2,-0.5). The cubic increases without bound as x increases and decreases without bound as x decreases. The reciprocal approaches y = 0 as x becomes large and has vertical asymptote x = 0.

Reason: Substitute values and observe signs. The cubic has no turning point in the plotted interval; the reciprocal branches lie in opposite quadrants from y = 1/x because of the negative sign.

Check: Verify reciprocal products xy = -1 and cubic sums y = x^3 + x; check that no branch crosses x = 0.

Compare Cubic and Reciprocal Graph Shapes: Paired Plotting Worksheet

Original tasks 4 / complete keys

Task 4

Compare y = x^3 - 2x and y = 3/x using x-values -3, -1, 1, 3. State intercepts and asymptotes.

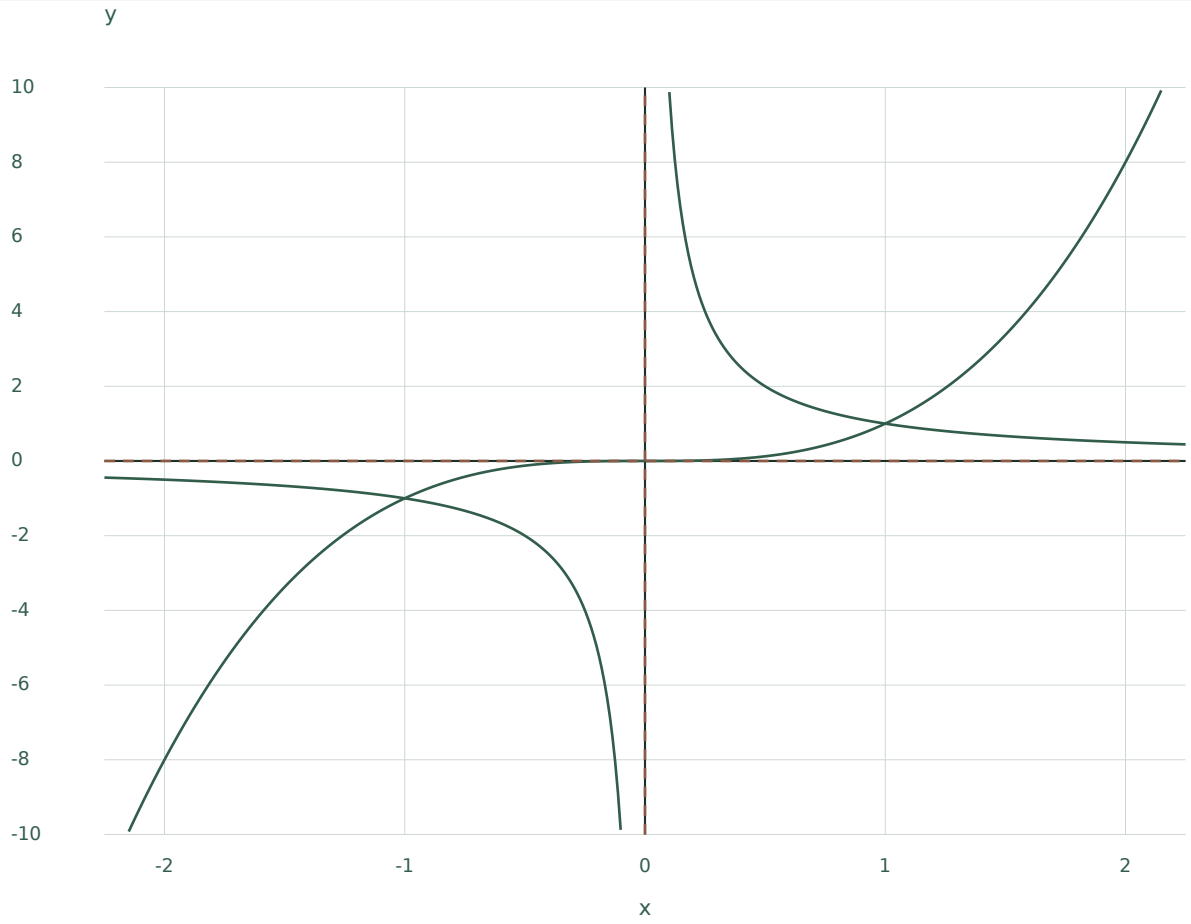
Answer: Cubic points: (-3,-21), (-1,1), (1,-1), (3,21). Reciprocal points: (-3,-1), (-1,-3), (1,3), (3,1). The cubic crosses the x-axis at approximately -1.41, 0, and 1.41. The reciprocal has asymptotes x = 0 and y = 0 and no x-intercept.

Reason: Compute each y-value by substitution. The cubic has three x-intercepts because x^3 - 2x = x(x^2 - 2). The reciprocal is undefined at zero and never equals zero.

Check: Check cubic intercepts by solving x(x^2 - 2) = 0 and reciprocal values by xy = 3; confirm asymptote equations.

Compare Cubic and Reciprocal Graph Shapes: Paired Plotting Worksheet

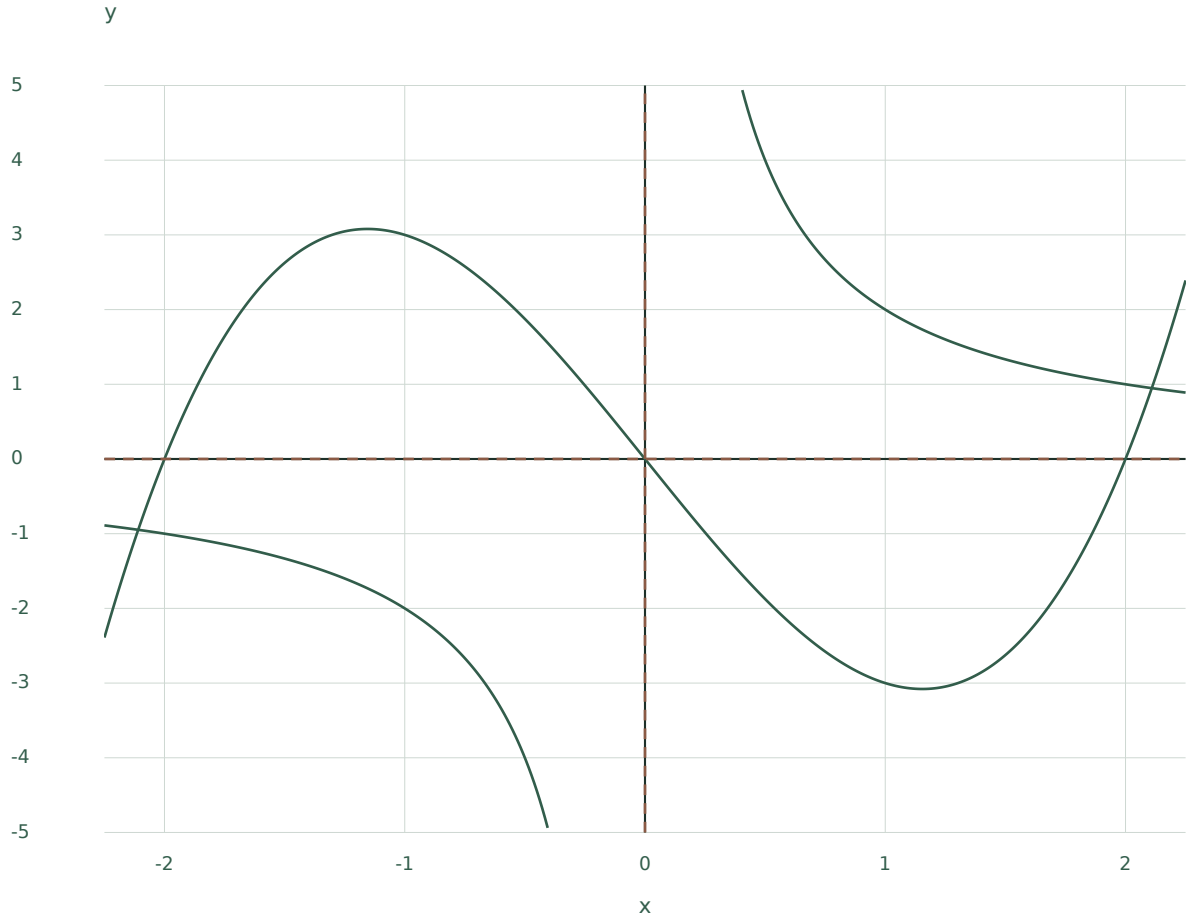
Task 1 / graph key



Green paths are the cubic and the two reciprocal branches. Brown dashed axes mark reciprocal asymptotes; they are not part of either function.

Compare Cubic and Reciprocal Graph Shapes: Paired Plotting Worksheet

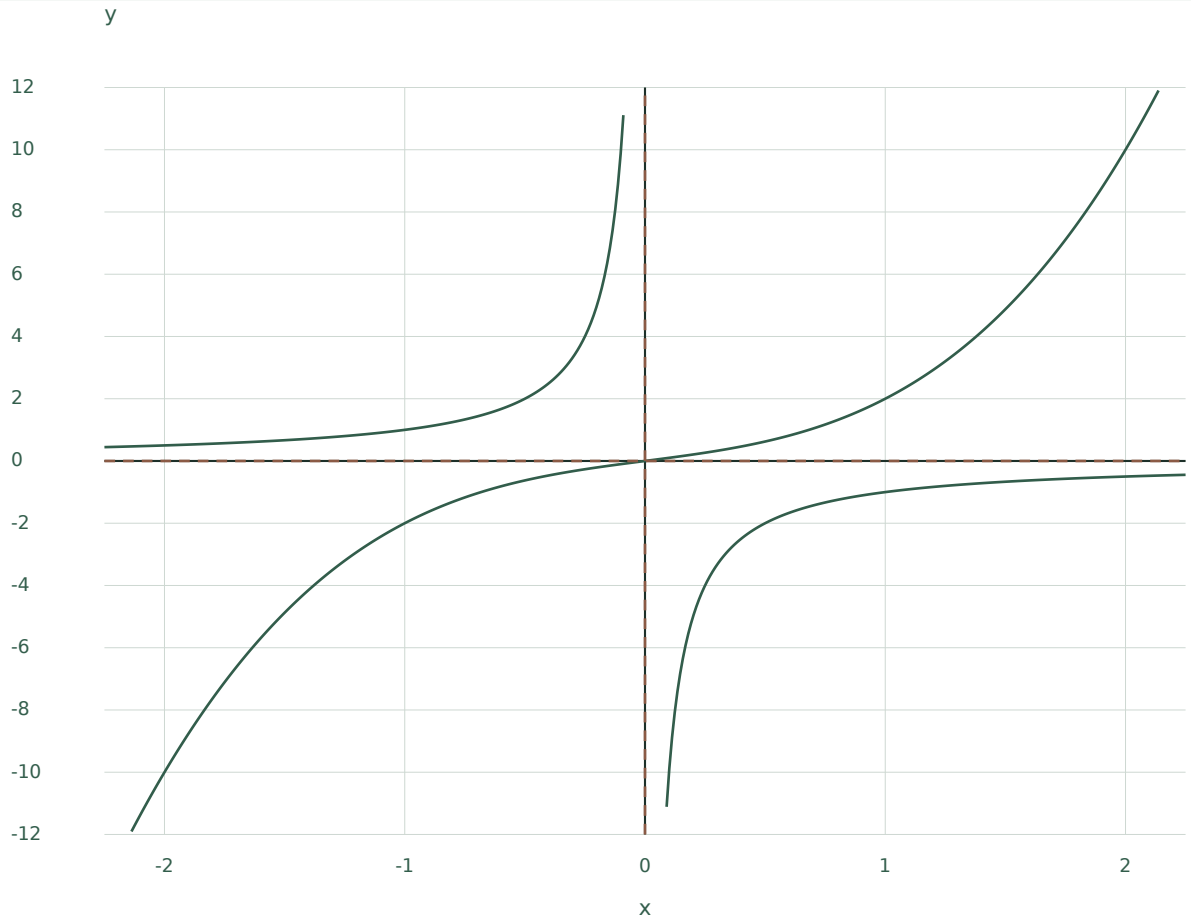
Task 2 / graph key



Green paths are the cubic and the two reciprocal branches. Brown dashed axes mark reciprocal asymptotes; they are not part of either function.

Compare Cubic and Reciprocal Graph Shapes: Paired Plotting Worksheet

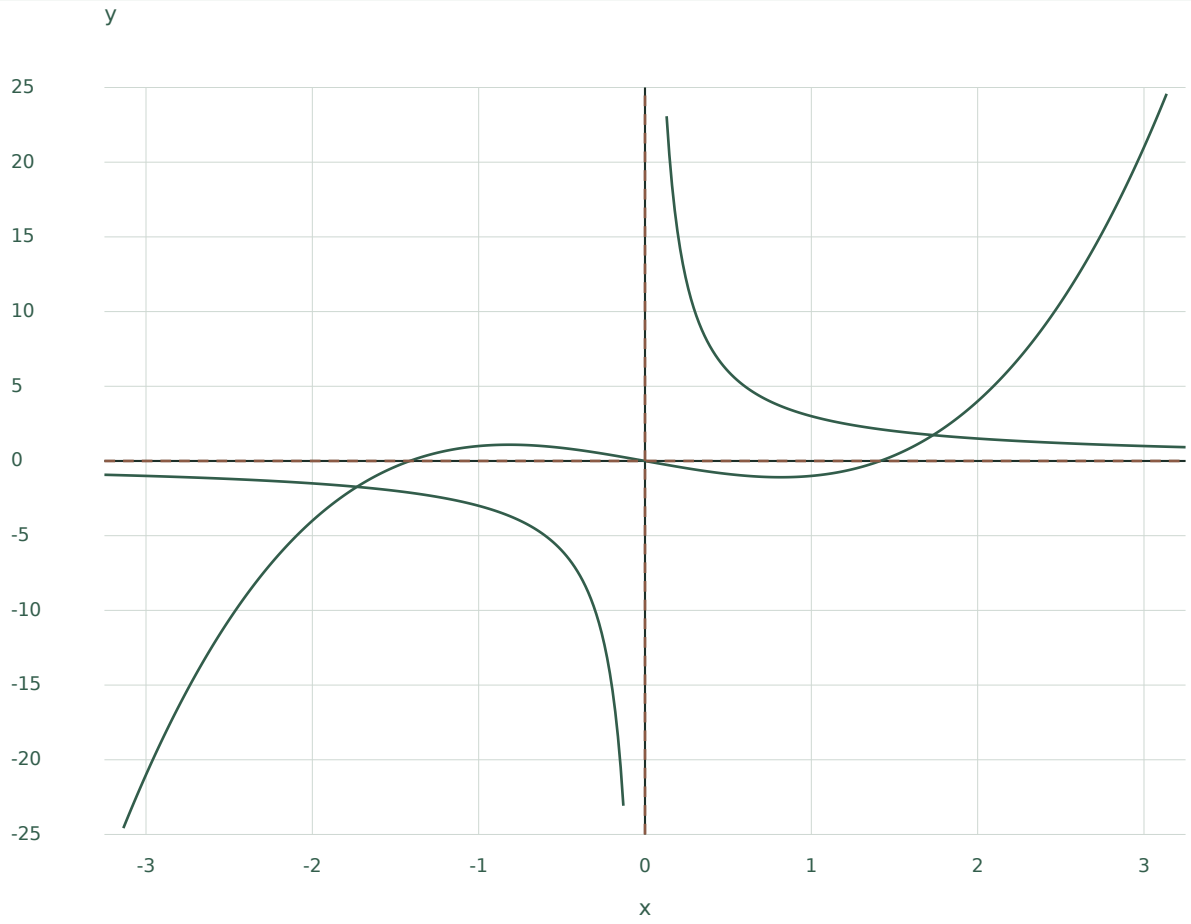
Task 3 / graph key



Green paths are the cubic and the two reciprocal branches. Brown dashed axes mark reciprocal asymptotes; they are not part of either function.

Compare Cubic and Reciprocal Graph Shapes: Paired Plotting Worksheet

Task 4 / graph key



Green paths are the cubic and the two reciprocal branches. Brown dashed axes mark reciprocal asymptotes; they are not part of either function.

Compare Cubic and Reciprocal Graph Shapes: Paired Plotting Worksheet

Source rubric / reflection

- [ ] Plots at least five labeled points per curve or branch with correct coordinates.
- [ ] Draws smooth curves without connecting across a vertical asymptote.
- [ ] Labels vertical and horizontal asymptotes with dashed lines and equations.
- [ ] Identifies cubic turning points and states domain and range exclusions.
- [ ] Writes a clear comparison of shape, intercepts, and end behavior.
- [ ] Accepts valid alternative point choices and handwriting styles if coordinates and labels are correct.

Reflection: What did you check, what did you change, and what will you try next?