Drawing Reciprocal Graphs with Asymptotes: A Step-by-Step Worksheet Guide

Rubric key

Rubric accepts valid alternative point choices and smooth curve styles if asymptote behavior and sign pattern are correct.

Asymptotes correctly identified and drawn as dashed lines at x = h and y = k.

Value table includes at least two x-values on each side of h, with correct arithmetic.

Two separate branches drawn, one on each side of the vertical asymptote, without crossing either asymptote.

Branch positions match the sign of a relative to y = k.

At least two plotted points visibly lie on the drawn curves.

Labels distinguish asymptotes from the curve; alternative valid point choices and smooth curve styles are accepted if the asymptote behavior and sign pattern are correct.

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Complete worked example / separate answer key

Draw the graph of y = 2/(x - 1) + 3. Mark the asymptotes and plot points for x = -1, 0, 0.5, 1.5, 2, and 4.

Answer: Vertical asymptote: x = 1. Horizontal asymptote: y = 3. Since a = 2 > 0, the right branch is above y = 3 and the left branch is below y = 3. Computed points: x = -1 gives y = 2/(-2) + 3 = 2; x = 0 gives y = 2/(-1) + 3 = 1; x = 0.5 gives y = 2/(-0.5) + 3 = -1; x = 1.5 gives y = 2/(0.5) + 3 = 7; x = 2 gives y = 2/(1) + 3 = 5; x = 4 gives y = 2/(3) + 3 = 3.67 (to two decimal places). Plot these points, draw dashed lines at x = 1 and y = 3, and connect the left points (-1, 2), (0, 1), (0.5, -1) with a smooth branch approaching x = 1 vertically and approaching y = 3 as x → -∞, and the right points (1.5, 7), (2, 5), (4, 3.67) with a second branch approaching x = 1 vertically and approaching y = 3 as x → +∞. Check: re-substitute x = 0.5 to confirm y = -1, and confirm the left-branch y-values are all below 3 while the right-branch y-values are all above 3; no point is plotted at x = 1.

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Original tasks 1, 2 / complete keys

Task 1

Draw $y = 1/(x - 2) + 1$. Mark the asymptotes and plot points for $x = 0$, 1, 1.5, 2.5, 3, and 5.

Answer: Vertical asymptote $x = 2$; horizontal asymptote $y = 1$. Points: (0, 0.5), (1, 0), (1.5, -1), (2.5, 3), (3, 2), (5, 1.33). Left branch below $y = 1$, right branch above $y = 1$. Draw two separate smooth branches, one on each side of $x = 2$, approaching the dashed asymptotes without crossing them.

Reason: Substitute each x into $y = 1/(x - 2) + 1$. Since $a = 1 > 0$, the right branch is above $y = 1$ and the left branch is below. The two branches approach $x = 2$ and $y = 1$ without crossing.

Check: Confirm each computed y by re-substituting, verify that left-branch y-values are less than 1 and right-branch y-values are greater than 1, and check that no plotted point lies at $x = 2$.

Task 2

Draw $y = -2/(x + 1) + 4$. Mark the asymptotes and plot points for $x = -3$, -2, -1.5, -0.5, 0, and 2.

Answer: Vertical asymptote $x = -1$; horizontal asymptote $y = 4$. Points: (-3, 5), (-2, 6), (-1.5, 8), (-0.5, 0), (0, 2), (2, 3.33). Left branch above $y = 4$, right branch below $y = 4$. Draw two separate smooth branches, one on each side of $x = -1$, approaching the dashed asymptotes without crossing them.

Reason: Here $h = -1$ and $k = 4$. Since $a = -2 < 0$, the branch on the right of $x = -1$ is below $y = 4$ and the branch on the left is above $y = 4$. Compute each y carefully, noting that $x + 1$ is negative for $x < -1$.

Check: Recompute one point on each side, confirm left-branch y-values exceed 4 and right-branch y-values are less than 4, and verify the branches do not cross $x = -1$ or $y = 4$.

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Original tasks 3, 4 / complete keys

Task 3

Draw y = 3/(x - 0) - 2, that is y = 3/x - 2. Mark the asymptotes and plot points for x = -3, -1, -0.5, 0.5, 1, and 3.

Answer: Vertical asymptote x = 0 (the y-axis); horizontal asymptote y = -2. Points: (-3, -3), (-1, -5), (-0.5, -8), (0.5, 4), (1, 1), (3, -1). Left branch below y = -2, right branch above y = -2. Draw two separate smooth branches, one on each side of the y-axis, approaching the dashed asymptotes without crossing them.

Reason: With h = 0 and k = -2, the vertical asymptote is the y-axis. Since a = 3 > 0, the right branch is above y = -2 and the left branch is below. The values near x = 0 have large magnitude, showing the steep approach.

Check: Check that x = 0 is not plotted, confirm the sign pattern relative to y = -2, and re-substitute two points to verify arithmetic.

Task 4

Draw y = -1/(x - 3) + 0, that is y = -1/(x - 3). Mark the asymptotes and plot points for x = 1, 2, 2.5, 3.5, 4, and 6.

Answer: Vertical asymptote x = 3; horizontal asymptote y = 0 (the x-axis). Points: (1, 0.5), (2, 1), (2.5, 2), (3.5, -2), (4, -1), (6, -0.33). Left branch above y = 0, right branch below y = 0. Draw two separate smooth branches, one on each side of x = 3, approaching the dashed asymptotes without crossing them.

Reason: Here k = 0, so the horizontal asymptote is the x-axis. Since a = -1 < 0, the right branch is below y = 0 and the left branch is above y = 0. Compute each y and note the sign change across x = 3.

Check: Verify that left-branch y-values are positive and right-branch y-values are negative, confirm no point lies at x = 3, and re-substitute one point from each branch.

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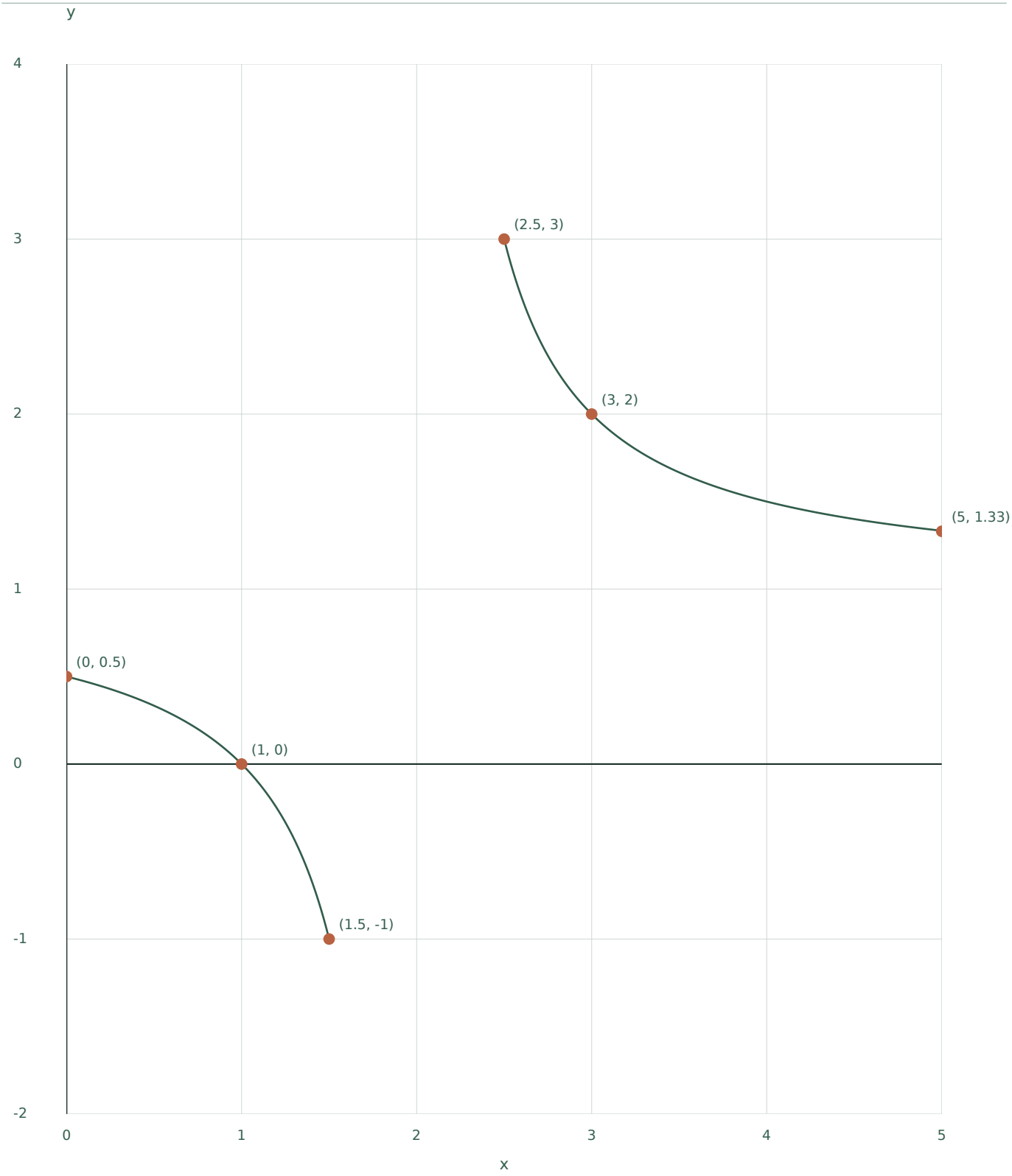
Task 1, Task 2, Task 3, Task 4 / Computed value tables and check notes

One valid plotting style; alternative correct point choices and smooth curve styles are acceptable.

Exercise	Equation	Asymptotes	Points (x, y)
1	y = 1/(x - 2) + 1	x = 2; y = 1	(0, 0.5), (1, 0), (1.5, -1), (2.5, 3), (3, 2), (5, 1.33)
2	y = -2/(x + 1) + 4	x = -1; y = 4	(-3, 5), (-2, 6), (-1.5, 8), (-0.5, 0), (0, 2), (2, 3.33)
3	y = 3/x - 2	x = 0; y = -2	(-3, -3), (-1, -5), (-0.5, -8), (0.5, 4), (1, 1), (3, -1)
4	y = -1/(x - 3)	x = 3; y = 0	(1, 0.5), (2, 1), (2.5, 2), (3.5, -2), (4, -1), (6, -0.33)

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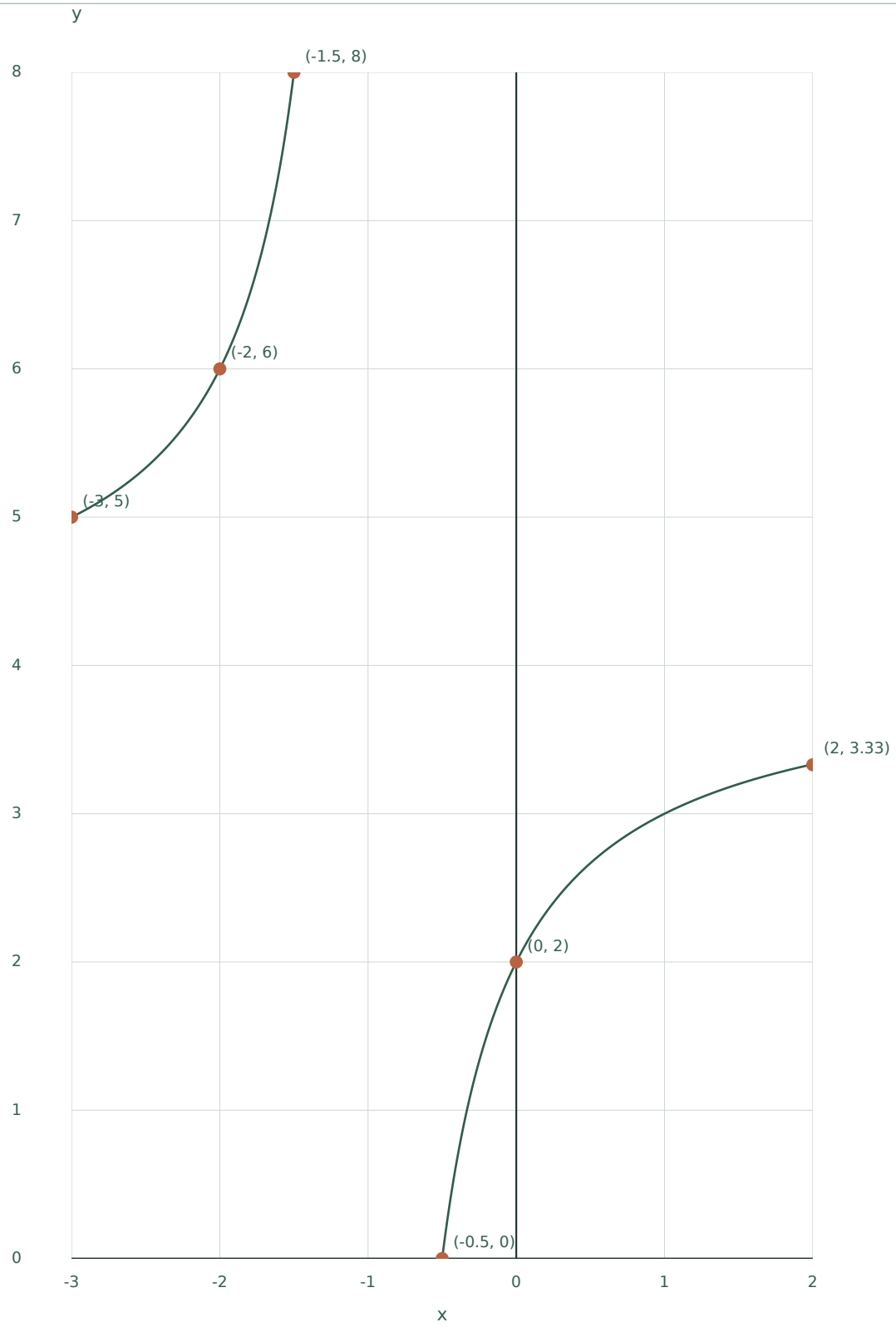
Task 1 / Exercise 1 answer: y = 1/(x - 2) + 1



Vertical asymptote x = 2; horizontal asymptote y = 1. Left branch below y = 1; right branch above y = 1. No point at x = 2.

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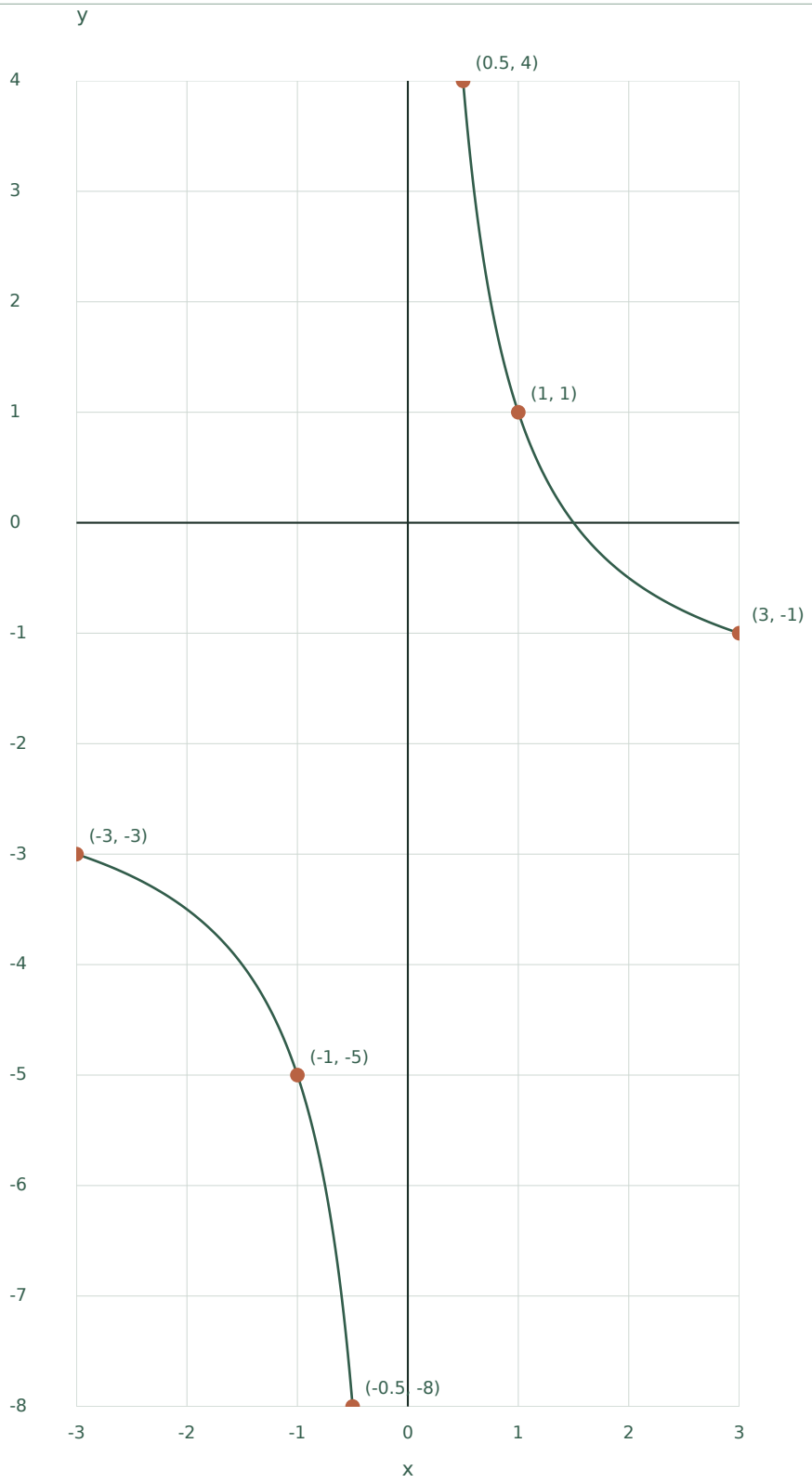
Task 2 / Exercise 2 answer: y = -2/(x + 1) + 4



Vertical asymptote x = -1; horizontal asymptote y = 4. Left branch above y = 4; right branch below y = 4. No point at x = -1.

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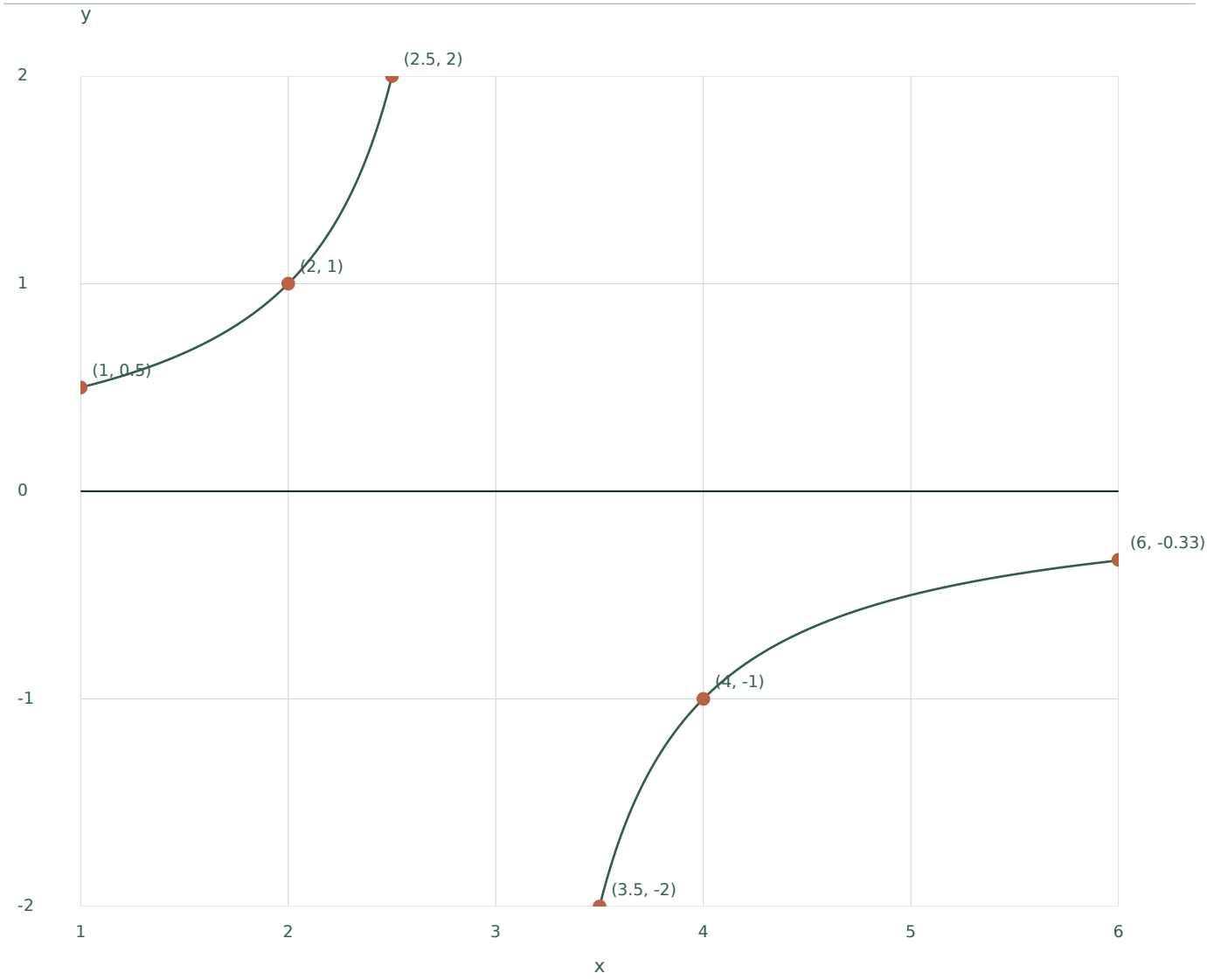
Task 3 / Exercise 3 answer: y = 3/x - 2



Vertical asymptote x = 0 (y-axis); horizontal asymptote y = -2. Left branch below y = -2; right branch above y = -2. No point at x = 0.

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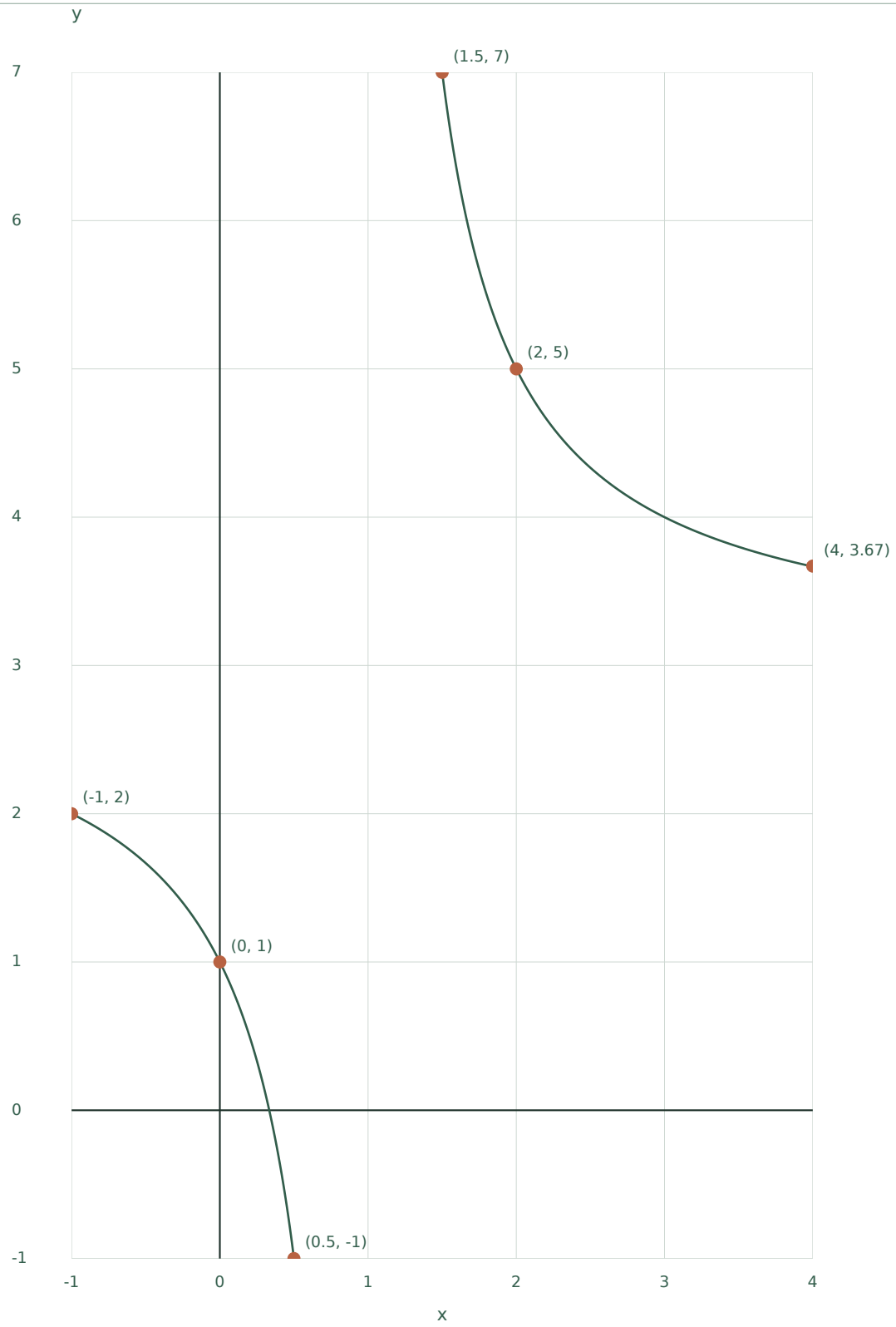
Task 4 / Exercise 4 answer: y = -1/(x - 3)



Vertical asymptote x = 3; horizontal asymptote y = 0 (x-axis). Left branch above y = 0; right branch below y = 0. No point at x = 3.

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Worked example / Worked example: y = 2/(x - 1) + 3



Dashed vertical asymptote x = 1 and horizontal asymptote y = 3. Two separate branches; no point at x = 1.

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Source rubric / reflection

- [ ] Asymptotes correctly identified and drawn as dashed lines at x = h and y = k.
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- [ ] Two separate branches drawn, one on each side of the vertical asymptote, without crossing either asymptote.
- [ ] Branch positions match the sign of a relative to y = k.
- [ ] At least two plotted points visibly lie on the drawn curves.
- [ ] Labels distinguish asymptotes from the curve; alternative valid point choices and smooth curve styles are accepted if the asymptote behavior and sign pattern are correct.

Reflection: What did you check, what did you change, and what will you try next?