Graph Factored Polynomials: Intercepts, Multiplicity, and End Behavior

Rubric key

Qualitative rubric; valid alternatives accepted.

All distinct real roots are plotted at correct x-values.

Each intercept is labeled cross or touch according to multiplicity parity.

End behavior matches degree parity and leading coefficient sign.

Sign table intervals match the curve's position above or below the axis.

y-intercept is correct and plotted.

Curve is smooth with no more than degree minus one turning points.

Graph Factored Polynomials: Intercepts, Multiplicity, and End Behavior

Complete worked example / separate answer key

Sketch f(x) = -(x + 2)(x - 1)^2. Identify roots, multiplicities, end behavior, y-intercept, and interval signs.

Answer: Roots: x = -2 with multiplicity 1 (cross), x = 1 with multiplicity 2 (touch and turn). Degree 3, leading coefficient -1, so as x -> -inf, f -> +inf; as x -> +inf, f -> -inf. y-intercept: f(0) = -(2)(1) = -2. Sign table: for x < -2, choose -3: -(-1)(16) = +16, positive. For -2 < x < 1, choose 0: -2, negative. For x > 1, choose 2: -(4)(1) = -4, negative. Sketch: left end rises, crosses at -2 downward, stays below axis, touches at 1 and turns back below, then falls to the right. Smooth curve with at most two turning points.

Graph Factored Polynomials: Intercepts, Multiplicity, and End Behavior

Original tasks 1, 2, 3 / complete keys

Task 1

For f(x) = (x - 3)(x + 1)^2, list roots with multiplicities, state end behavior, and find the y-intercept.

Answer: Roots: x = 3 multiplicity 1 (cross); x = -1 multiplicity 2 (touch). Degree 3, leading coefficient 1: x -> -inf, f -> -inf; x -> +inf, f -> +inf. y-intercept: f(0) = (-3)(1) = -3.

Reason: Set each factor to zero. Add exponents for degree 3. Positive leading coefficient with odd degree gives fall-left, rise-right. Evaluate at x = 0.

Check: Verify degree by adding exponents, confirm parity at each root, and recompute f(0) = (0-3)(0+1)^2 = -3.

Task 2

For g(x) = -2(x - 1)^3(x + 4), state degree, leading coefficient, end behavior, and the behavior at each intercept.

Answer: Degree 4, leading coefficient -2. As x -> -inf, g -> -inf; as x -> +inf, g -> -inf. At x = 1 multiplicity 3: cross with flattening. At x = -4 multiplicity 1: cross straight.

Reason: Multiplicities 3 and 1 sum to 4. Leading coefficient is -2 times 1 times 1 = -2. Even degree with negative leading coefficient falls on both ends. Odd multiplicities cross.

Check: Confirm degree 4 and negative leading coefficient give both ends down. Check that both multiplicities are odd, so both intercepts cross.

Task 3

Build a sign table for h(x) = (x + 3)(x - 2)^2 and state whether the graph is above or below the axis on each interval.

Answer: Roots at -3 and 2. Intervals: x < -3: negative; -3 < x < 2: positive; x > 2: positive. The graph is below the axis left of -3, above between -3 and 2, and above right of 2.

Reason: Test x = -4: (-1)(36) = -36 negative. Test x = 0: (3)(4) = 12 positive. Test x = 3: (6)(1) = 6 positive. The squared factor is never negative, so sign changes only at -3.

Check: Re-evaluate one test point per interval and confirm the squared factor's sign is always nonnegative.

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Original tasks 4 / complete keys

Task 4

Sketch k(x) = (x - 1)^2(x + 2)^2. Describe roots, multiplicities, end behavior, y-intercept, and interval signs without drawing an image.

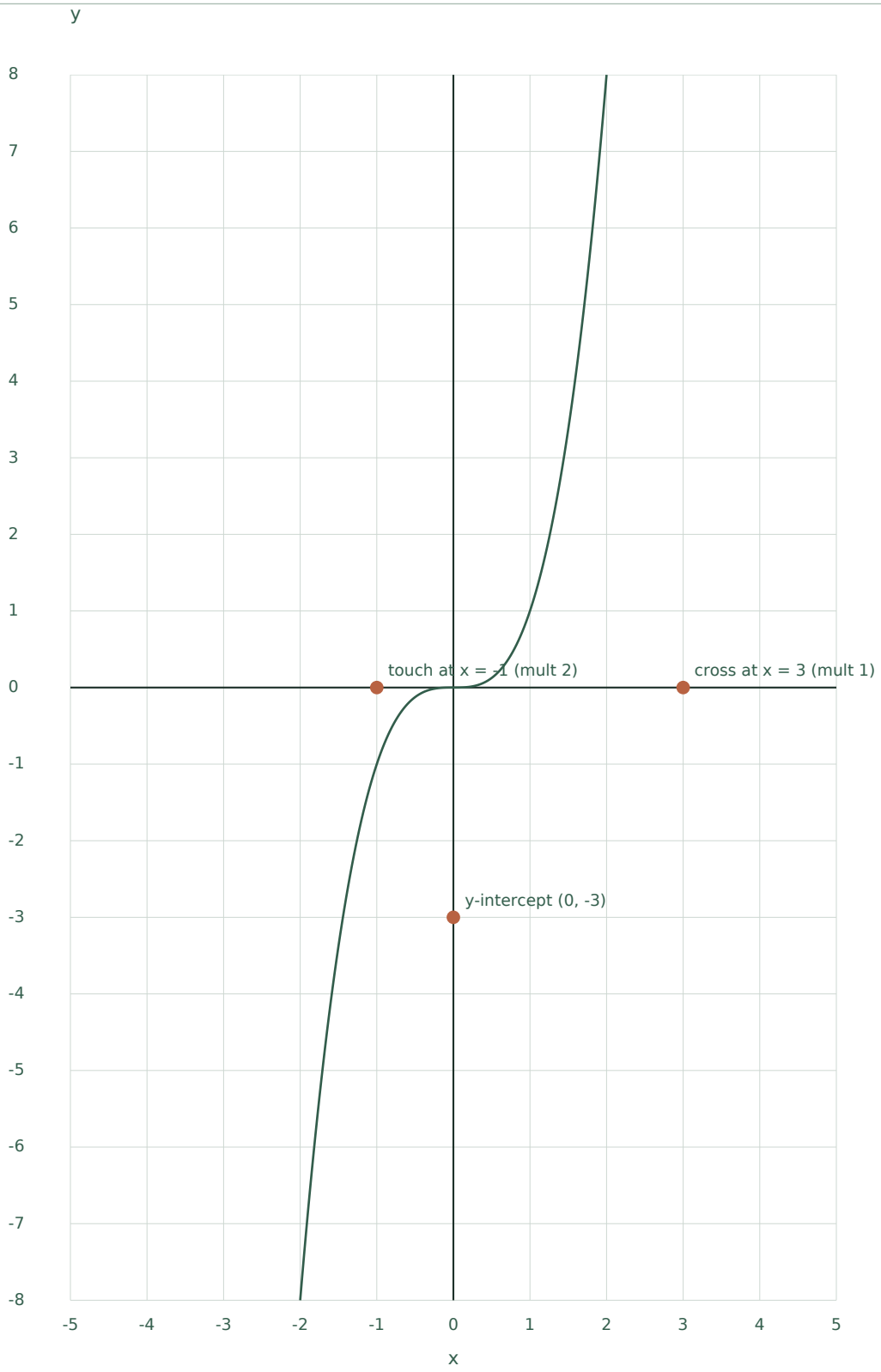
Answer: Roots: x = 1 multiplicity 2 (touch), x = -2 multiplicity 2 (touch). Degree 4, leading coefficient 1: both ends rise. y-intercept: k(0) = (1)(4) = 4. Sign table: positive for x < -2, positive for -2 < x < 1, positive for x > 1. The curve stays above the axis except touching at -2 and 1, with a smooth dip between touches.

Reason: Both factors are squared, so the polynomial is nonnegative everywhere. Even degree with positive leading coefficient rises on both ends. Evaluate at zero for the y-intercept.

Check: Confirm both multiplicities are even, degree is 4, leading coefficient positive, and all interval signs are positive. Check y-intercept equals 4.

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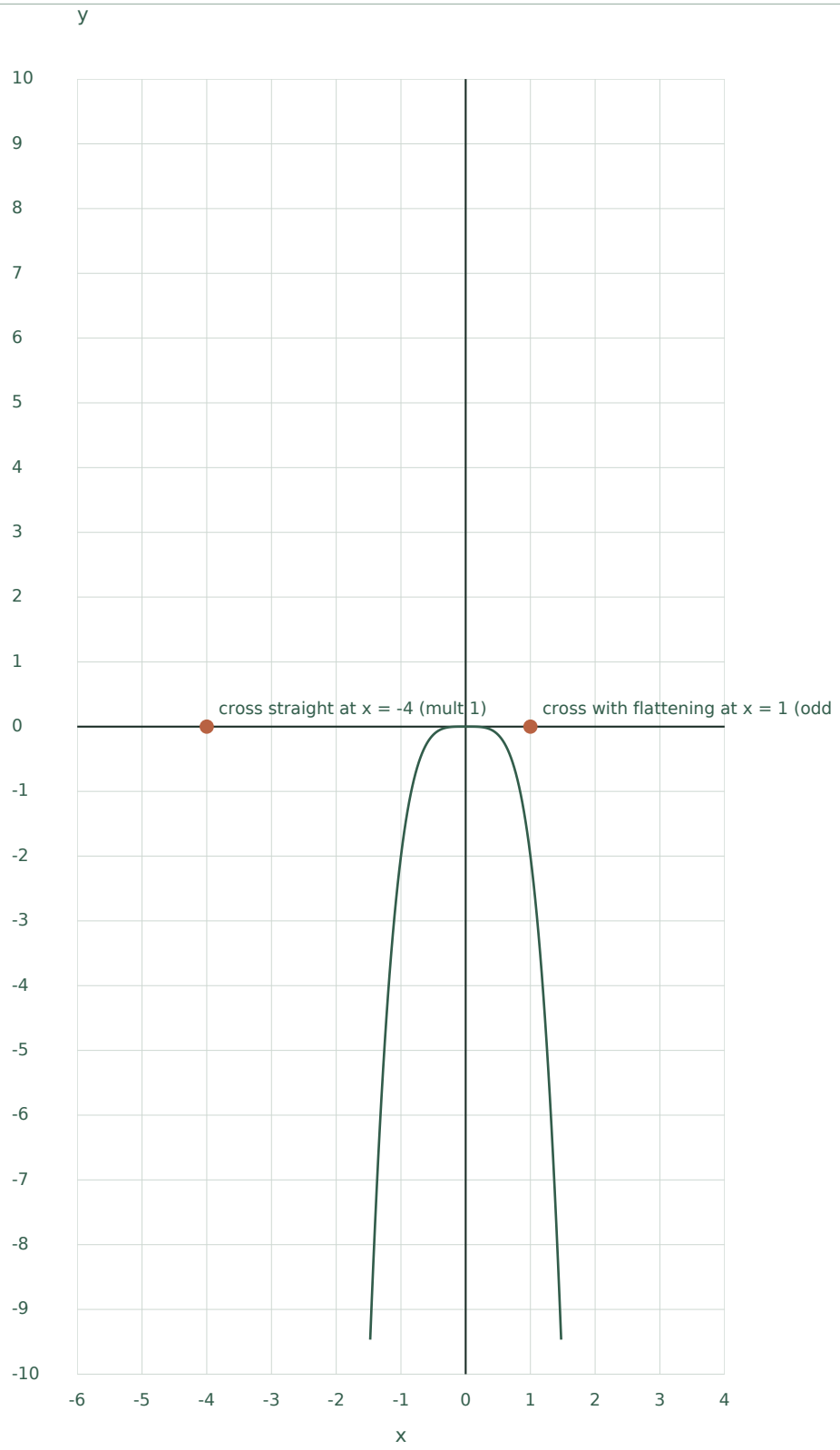
Task 1 / Separate answer key: qualitative sketches and analysis



Qualitative sketch; valid alternatives with different heights but same structural features are accepted.

Graph Factored Polynomials: Intercepts, Multiplicity, and End Behavior

Task 2 / Answer key: Exercise 2 qualitative sketch



Qualitative sketch; valid alternatives with different heights but same structural features are accepted.

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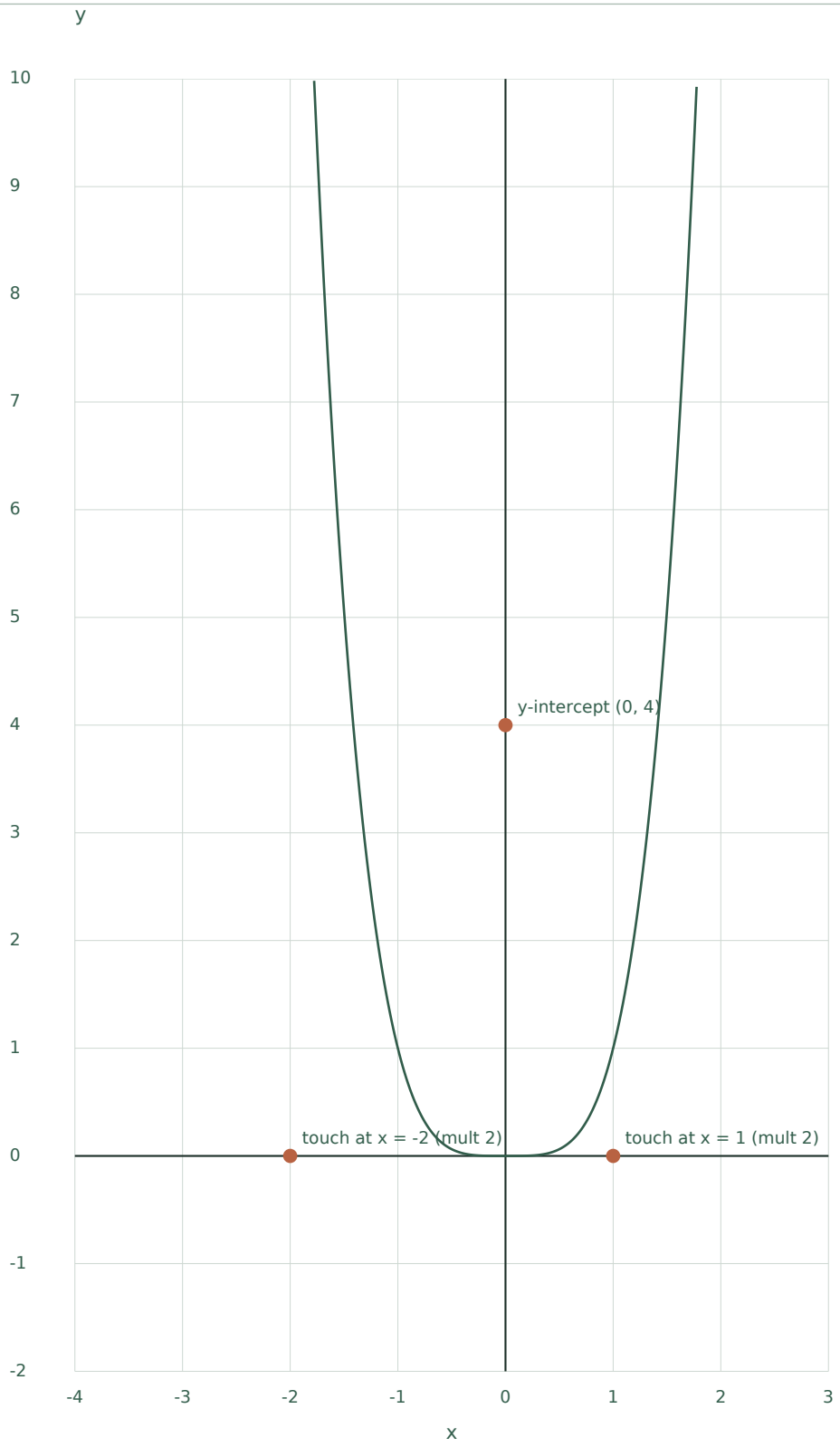
Task 3 / Answer key: Exercise 3 sign table

Signs only; qualitative.

Interval	Test x-value	Sign of f(x)	Position relative to axis
$x < -3$	-4	negative	below
$-3 < x < 2$	0	positive	above
$x > 2$	3	positive	above

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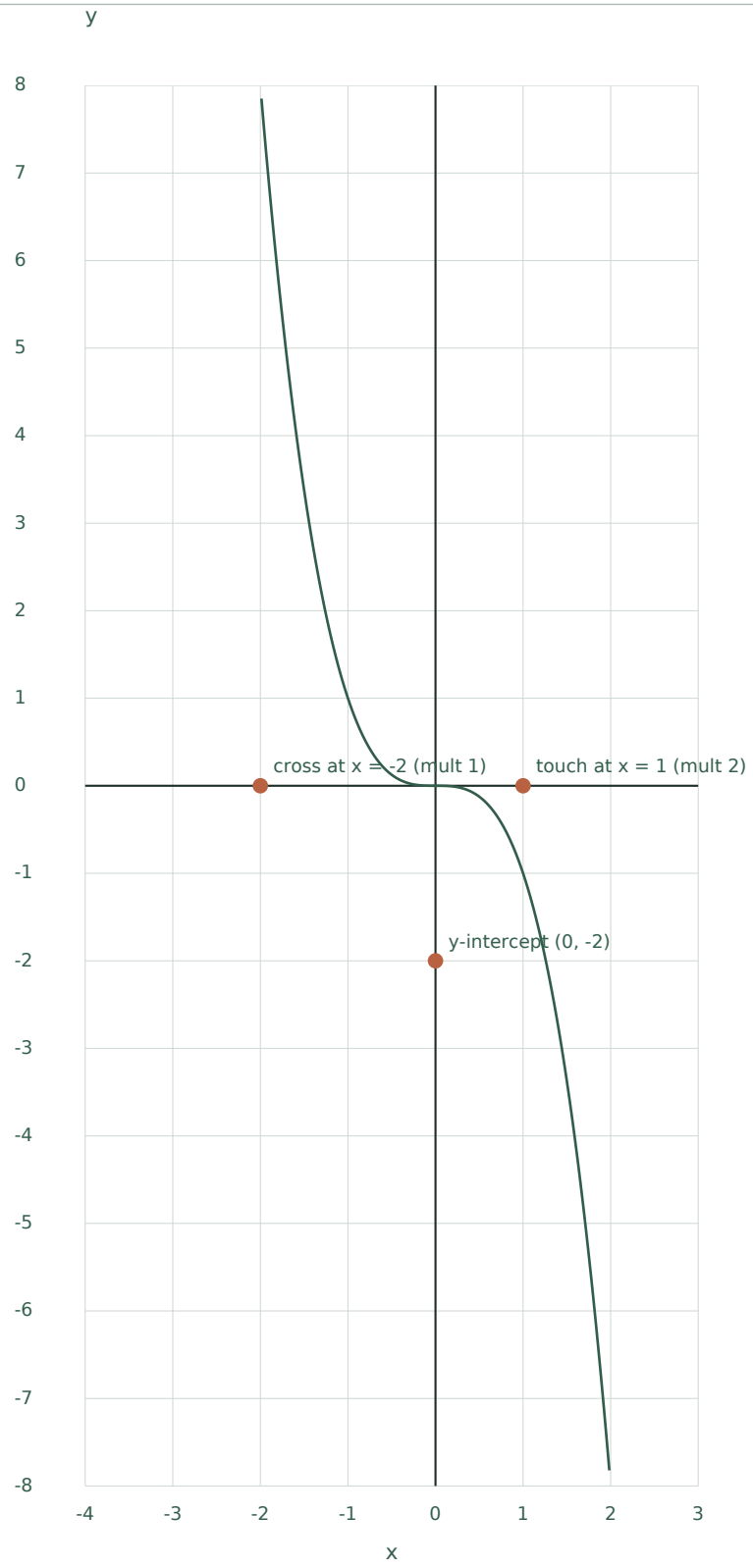
Task 4 / Answer key: Exercise 4 qualitative sketch



Qualitative sketch; valid alternatives with different heights but same structural features are accepted.

Graph Factored Polynomials: Intercepts, Multiplicity, and End Behavior

Worked example / Worked example: f(x) = -(x + 2)(x - 1)^2



Qualitative sketch; valid alternatives with different heights but same structural features are accepted.

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Source rubric / reflection

- [ ] All distinct real roots are plotted at correct x-values.
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- [ ] End behavior matches degree parity and leading coefficient sign.
- [ ] Sign table intervals match the curve's position above or below the axis.
- [ ] y-intercept is correct and plotted.
- [ ] Curve is smooth with no more than degree minus one turning points.
- [ ] Valid alternative sketches with different heights but same structural features are accepted.

Reflection: What did you check, what did you change, and what will you try next?