Graph Linear Programming Constraints and Shade the Feasible Region

Rubric

Rubric criteria for grading the linear programming work.

Boundary lines are drawn accurately with correct solid or dashed style.

Shading is applied to the correct side of each constraint.

The feasible region is clearly identified as the overlap.

All vertices are listed with exact coordinates.

The objective function is evaluated at every vertex.

The optimal vertex and value are stated with correct units or context. Work is organized and readable, with labels and a consistent scale.

Graph Linear Programming Constraints and Shade the Feasible Region

Complete worked example / separate answer key

A workshop makes two products, A and B. Each unit of A uses 2 hours of labor and 1 unit of material; each unit of B uses 1 hour of labor and 2 units of material. There are 8 labor hours and 8 material units available. Profit is 3 per unit of A and 2 per unit of B. Maximize profit subject to $2x + y \leq 8$, $x + 2y \leq 8$, $x \geq 0$, $y \geq 0$, where x is units of A and y is units of B.

Answer: Graph the lines $2x + y = 8$ and $x + 2y = 8$. Test (0,0): both inequalities are true, so shade toward the origin. The feasible region is bounded by the axes and the two lines. Vertices: (0,0), (4,0), (0,4), and the intersection of $2x + y = 8$ and $x + 2y = 8$. Solving gives $x = 8/3$ and $y = 8/3$. Evaluate $P = 3x + 2y$: at (0,0), $P = 0$; at (4,0), $P = 12$; at (0,4), $P = 8$; at (8/3, 8/3), $P = 3(8/3) + 2(8/3) = 8 + 16/3 = 40/3 \approx 13.33$. The maximum profit is 40/3 at (8/3, 8/3).

Graph Linear Programming Constraints and Shade the Feasible Region

Original tasks 1, 2 / complete keys

Task 1

Maximize $P = 2x + 3y$ subject to $x + y \leq 4$, $x \geq 0$, $y \geq 0$. Graph the constraints, shade the feasible region, and find the optimal vertex and value.

Answer: Vertices: (0,0), (4,0), (0,4). P values: 0, 8, 12. Maximum is 12 at (0,4).

Reason: The line $x + y = 4$ has intercepts (4,0) and (0,4). Testing (0,0) gives $0 \leq 4$, so shade toward the origin. The feasible region is the triangle with vertices (0,0), (4,0), (0,4). Evaluate P at each vertex: (0,0) gives 0; (4,0) gives 8; (0,4) gives 12. The largest value is 12.

Check: Test an interior point such as (1,1): it satisfies $x + y \leq 4$. Confirm each vertex satisfies all constraints. Recalculate P at (0,4): $2(0) + 3(4) = 12$.

Task 2

Minimize $C = 4x + y$ subject to $x + y \geq 3$, $x \leq 4$, $y \leq 3$, $x \geq 0$, $y \geq 0$. Graph the constraints, shade the feasible region, and find the optimal vertex and value.

Answer: Vertices: (0,3), (3,0), (4,0), and (4,3). C values: at (0,3), C = 3; at (3,0), C = 12; at (4,0), C = 16; at (4,3), C = 19. Minimum is 3 at (0,3).

Reason: The line $x + y = 3$ has intercepts (3,0) and (0,3). Testing (0,0) gives $0 \geq 3$, which is false, so shade away from the origin. The constraints $x \leq 4$ and $y \leq 3$ bound the region on the right and top. The feasible region is a polygon with vertices (0,3), (3,0), (4,0), and (4,3). Evaluate C at each: (0,3) gives 3; (3,0) gives 12; (4,0) gives 16; (4,3) gives 19. The smallest value is 3.

Check: Test (1,2): $1 + 2 \geq 3$ is true, and $1 \leq 4$, $2 \leq 3$ hold. Confirm (0,3) satisfies all constraints. Recalculate C at (0,3): $4(0) + 3 = 3$.

Graph Linear Programming Constraints and Shade the Feasible Region

Original tasks 3, 4 / complete keys

Task 3

Maximize P = x + 2y subject to 2x + y ≤ 6, x + 2y ≤ 6, x ≥ 0, y ≥ 0. Graph the constraints, shade the feasible region, and find the optimal vertex and value.

Answer: Vertices: (0,0), (3,0), (0,3), and (2,2). P values: 0, 3, 6, 6. Maximum is 6, achieved at both (0,3) and (2,2), and along the segment between them.

Reason: The lines 2x + y = 6 and x + 2y = 6 intersect at (2,2). Testing (0,0) satisfies both inequalities, so shade toward the origin. The feasible region is a quadrilateral with vertices (0,0), (3,0), (2,2), (0,3). Evaluate P: (0,0) gives 0; (3,0) gives 3; (2,2) gives 6; (0,3) gives 6. The maximum is 6, with a tie along the edge from (0,3) to (2,2).

Check: Test (1,1): 2(1)+1=3 ≤ 6 and 1+2=3 ≤ 6. Confirm the intersection by solving 2x + y = 6 and x + 2y = 6: subtract to get x = y, then 3x = 6, so x = 2, y = 2. Recalculate P at (2,2): 2 + 4 = 6.

Task 4

Minimize C = 3x + 2y subject to x + y ≥ 2, x + 3y ≥ 3, x ≥ 0, y ≥ 0. Graph the constraints, shade the feasible region, and find the optimal vertex and value.

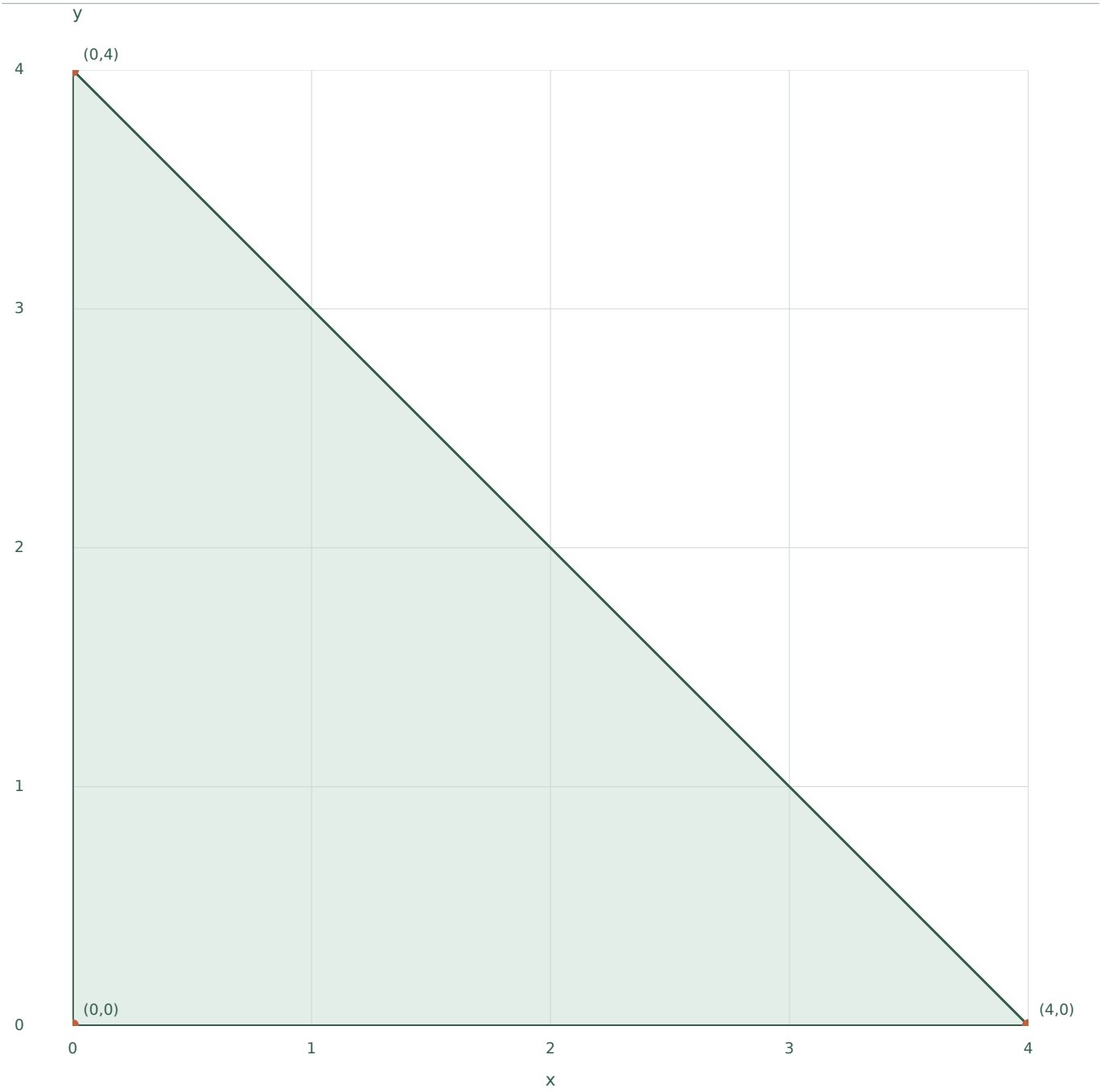
Answer: Vertices: (0,2), (3,0), and (1.5,0.5). C values: at (0,2), C = 4; at (3,0), C = 9; at (1.5,0.5), C = 5.5. Minimum is 4 at (0,2).

Reason: The line x + y = 2 has intercepts (2,0) and (0,2). Testing (0,0) gives 0 ≥ 2, false, so shade away from the origin. The line x + 3y = 3 has intercepts (3,0) and (0,1). Testing (0,0) gives 0 ≥ 3, false, so shade away from the origin. The feasible region is unbounded, with vertices (0,2), (3,0), and the intersection of x + y = 2 and x + 3y = 3. Solving gives x = 1.5, y = 0.5. Evaluate C: (0,2) gives 4; (3,0) gives 9; (1.5,0.5) gives 5.5. The minimum is 4.

Check: Test (1,1): 1+1=2 ≥ 2 and 1+3=4 ≥ 3. Confirm the intersection by substituting x = 2 - y into x + 3y = 3: 2 - y + 3y = 3, so 2y = 1, y = 0.5, x = 1.5. Recalculate C at (0,2): 3(0) + 2(2) = 4.

Graph Linear Programming Constraints and Shade the Feasible Region

Task 1 / Answer Key: Exercise 1



Feasible region is the triangle with vertices (0,0), (4,0), (0,4).

Graph Linear Programming Constraints and Shade the Feasible Region

Task 1 / Answer Key: Exercise 1 Values

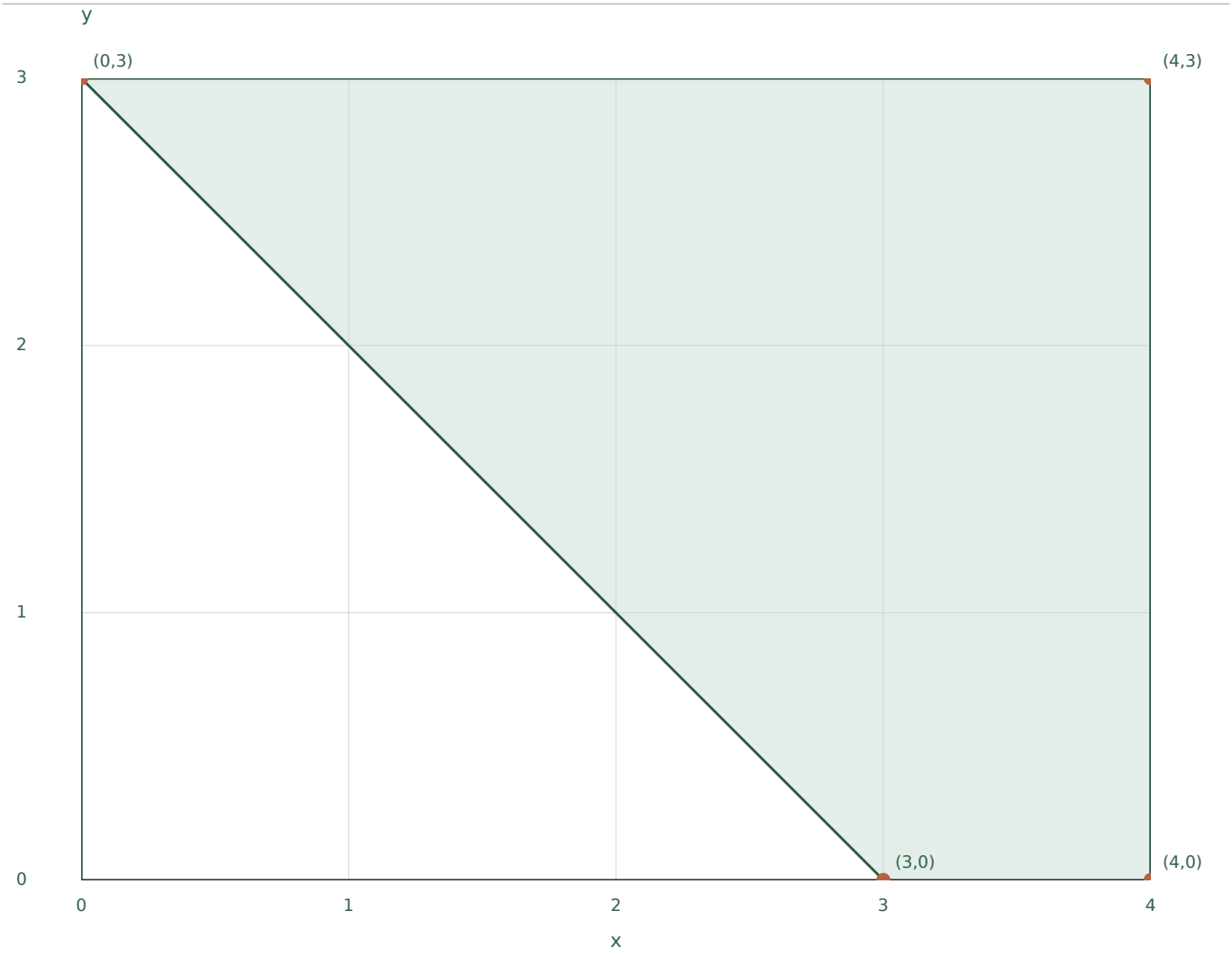
P values and optimal vertex for Exercise 1.

Vertices: (0,0), (4,0), (0,4). P values: 0, 8, 12. Maximum is 12 at (0,4).

Check: Test (1,1): $1 + 1 \leq 4$. Recalculate P at (0,4): $2(0) + 3(4) = 12$.

Graph Linear Programming Constraints and Shade the Feasible Region

Task 2 / Answer Key: Exercise 2



Feasible region is the polygon with vertices (0,3), (3,0), (4,0), (4,3).

Graph Linear Programming Constraints and Shade the Feasible Region

Task 2 / Answer Key: Exercise 2 Values

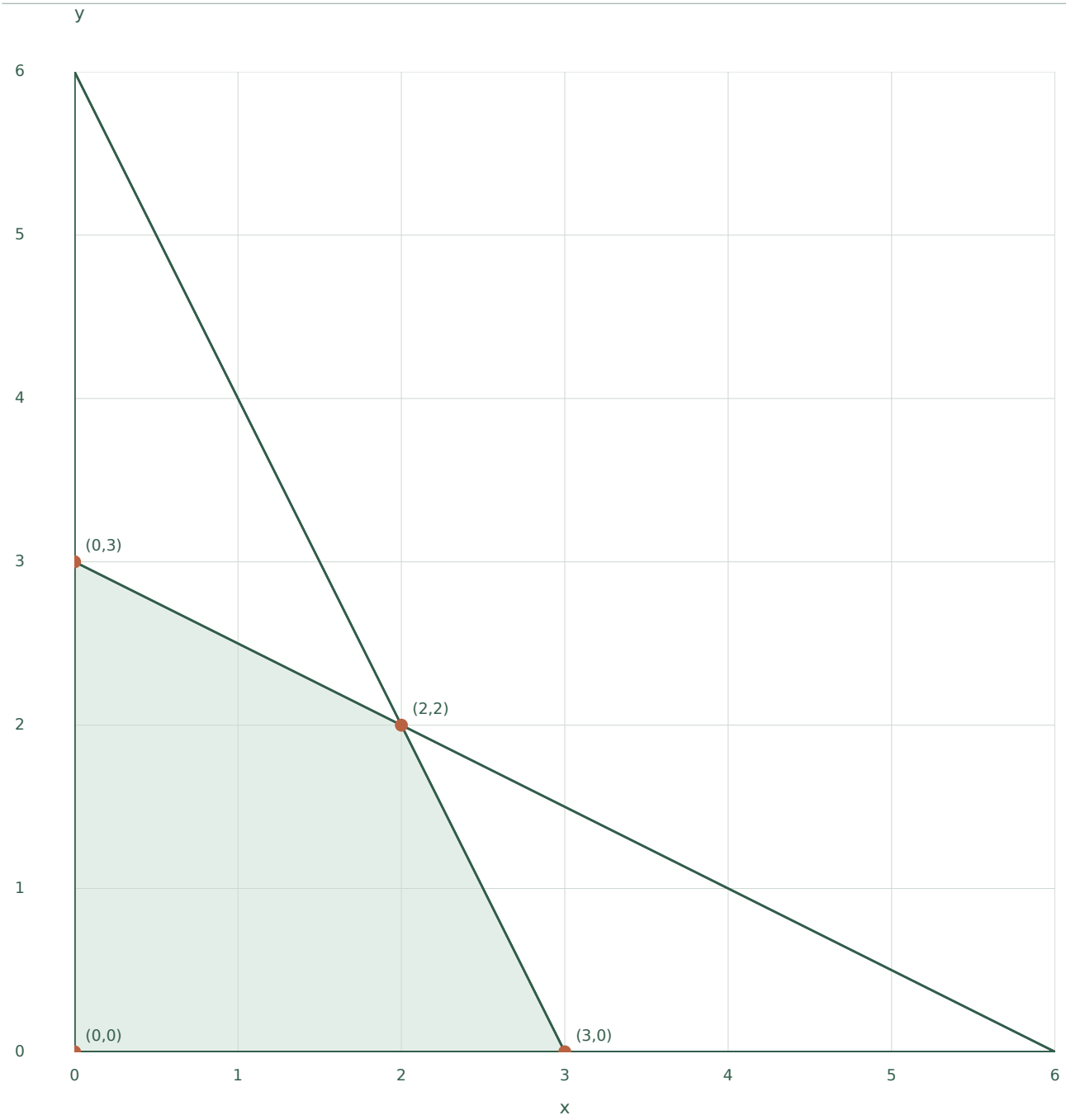
C values and optimal vertex for Exercise 2.

Vertices: (0,3), (3,0), (4,0), (4,3). C values: at (0,3), C = 3; at (3,0), C = 12; at (4,0), C = 16; at (4,3), C = 19. Minimum is 3 at (0,3).

Check: Test (1,2): $1 + 2 \geq 3$, $1 \leq 4$, $2 \leq 3$. Recalculate C at (0,3): $4(0) + 3 = 3$.

Graph Linear Programming Constraints and Shade the Feasible Region

Task 3 / Answer Key: Exercise 3



Feasible region is the quadrilateral with vertices (0,0), (3,0), (2,2), (0,3).

Graph Linear Programming Constraints and Shade the Feasible Region

Task 3 / Answer Key: Exercise 3 Values

P values and optimal vertices for Exercise 3.

Vertices: (0,0), (3,0), (0,3), (2,2). P values: 0, 3, 6, 6. Maximum is 6, achieved at both (0,3) and (2,2), and along the segment between them.

Check: Test (1,1): $2(1)+1=3 \leq 6$ and $1+2=3 \leq 6$. Intersection: solve $2x + y = 6$ and $x + 2y = 6$; subtract to get $x = y$, then $3x = 6$, so $x = 2$, $y = 2$. Recalculate P at (2,2): $2 + 4 = 6$.

Graph Linear Programming Constraints and Shade the Feasible Region

Task 4 / Answer Key: Exercise 4



Feasible region is unbounded, with vertices (0,2), (3,0), (1.5,0.5).

Graph Linear Programming Constraints and Shade the Feasible Region

Task 4 / Answer Key: Exercise 4 Values

C values and optimal vertex for Exercise 4.

Vertices: (0,2), (3,0), (1.5,0.5). C values: at (0,2), C = 4; at (3,0), C = 9; at (1.5,0.5), C = 5.5. Minimum is 4 at (0,2).

Check: Test (1,1): 1+1=2 ≥ 2 and 1+3=4 ≥ 3. Intersection: substitute x = 2 - y into x + 3y = 3: 2 - y + 3y = 3, so 2y = 1, y = 0.5, x = 1.5. Recalculate C at (0,2): 3(0) + 2(2) = 4.

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Worked example / Answer Key: Optimal Vertex and Value

Optimal vertex and value for the worked example.

The maximum profit is 40/3 at (8/3, 8/3).

Evaluate P = 3x + 2y at each vertex: (0,0) gives 0; (4,0) gives 12; (0,4) gives 8; (8/3, 8/3) gives 3(8/3) + 2(8/3) = 8 + 16/3 = 40/3 ≈ 13.33.

Graph Linear Programming Constraints and Shade the Feasible Region

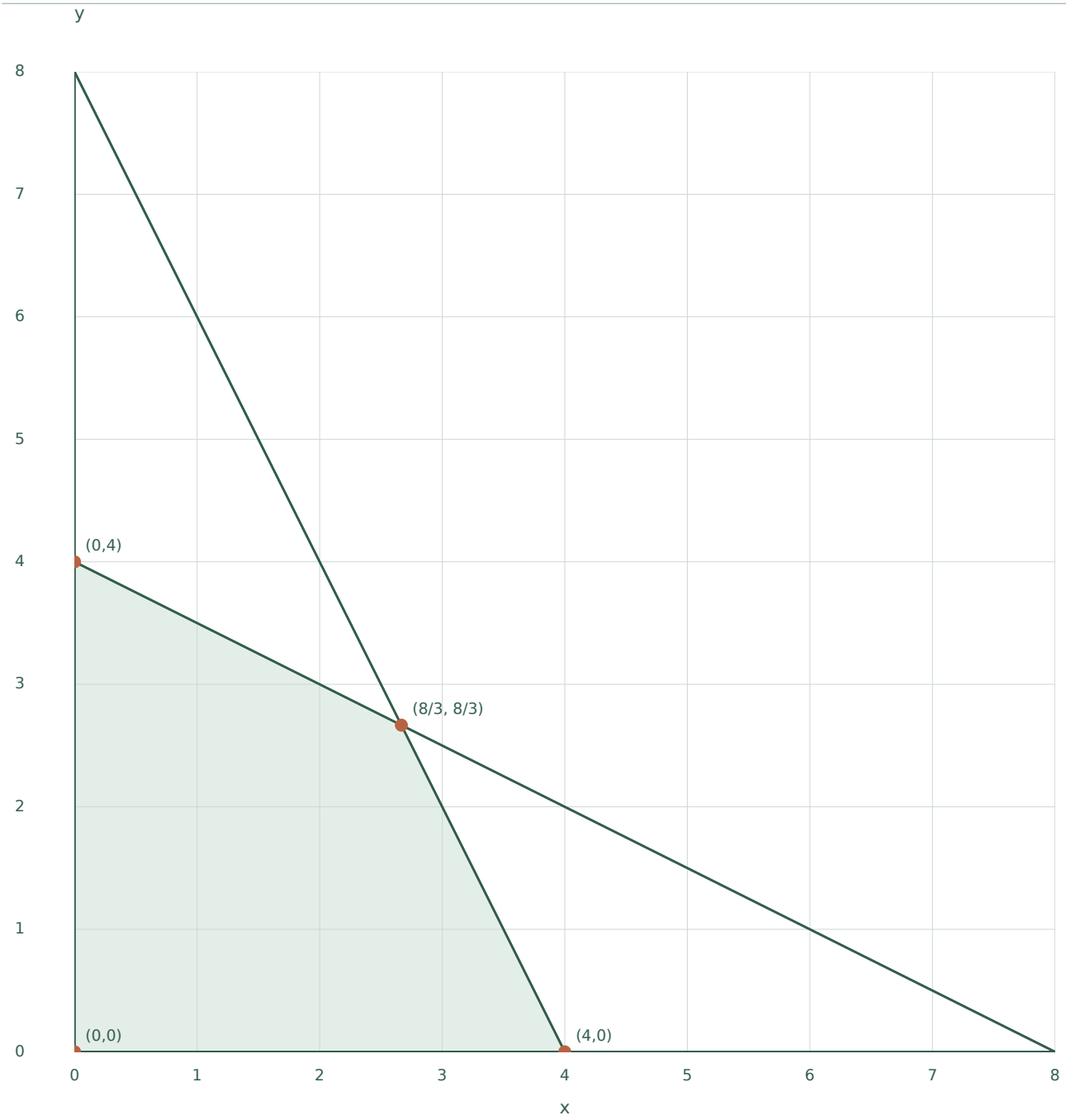
Worked example / Answer Key: Completed Vertex-Testing Table

Completed vertex-testing table for the worked example.

Vertex	Objective Value P = 3x + 2y	Decision
(0,0)	0	
(4,0)	12	
(0,4)	8	
(8/3, 8/3)	40/3 ≈ 13.33	Maximum

Graph Linear Programming Constraints and Shade the Feasible Region

Worked example / Answer Key: Worked Example Feasible Region and Vertices



Feasible region shaded toward origin. Vertices: (0,0), (4,0), (0,4), (8/3,8/3).

Graph Linear Programming Constraints and Shade the Feasible Region

Source rubric / reflection

- [ ] Boundary lines are drawn accurately with correct solid or dashed style.
- [ ] Shading is applied to the correct side of each constraint.
- [ ] The feasible region is clearly identified as the overlap.
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- [ ] The objective function is evaluated at every vertex.
- [ ] The optimal vertex and value are stated with correct units or context.
- [ ] Work is organized and readable, with labels and a consistent scale.

Reflection: What did you check, what did you change, and what will you try next?