Rectangle Optimization: Maximize Area and Minimize Perimeter

Original tasks 1, 2, 3 / complete keys

Task 1

A rectangle has a fixed perimeter of 20 meters. Find the dimensions that maximize area and state the maximum area.

Answer: Length 5 meters, width 5 meters, maximum area 25 square meters.

Reason: Constraint: 2L + 2W = 20, so W = 10 - L. Objective: A = L(10 - L) = 10L - L squared. This quadratic opens downward. Vertex at L = -10 divided by (2 times -1) = 5. Then W = 5. Area = 25.

Check: Test L = 4 and L = 6: areas are 24 and 24, both less than 25, confirming the maximum at L = 5.

Task 2

A rectangle must have an area of 36 square centimeters. Find the dimensions that minimize perimeter and state the minimum perimeter.

Answer: Length 6 centimeters, width 6 centimeters, minimum perimeter 24 centimeters.

Reason: Constraint: L times W = 36, so W = 36 divided by L. Objective: P = 2L + 2W = 2L + 72 divided by L. By symmetry or by testing, the minimum occurs at L = 6, giving W = 6 and P = 24. Alternatively, for fixed area the minimum perimeter rectangle is a square.

Check: Test L = 4 gives W = 9 and P = 26; L = 9 gives W = 4 and P = 26; both exceed 24.

Task 3

A rectangular garden uses a wall as one side and 30 meters of fencing for the other three sides. Find the dimensions that maximize area and state the maximum area.

Answer: Length 15 meters (parallel to the wall), width 7.5 meters, maximum area 112.5 square meters.

Reason: Constraint: L + 2W = 30, so L = 30 - 2W. Objective: A = L times W = (30 - 2W)W = 30W - 2W squared. This opens downward. Vertex at W = -30 divided by (2 times -2) = 7.5. Then L = 15. Area = 112.5.

Check: Test W = 7 gives L = 16 and area 112; W = 8 gives L = 14 and area 112; both less than 112.5.

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Original tasks 4 / complete keys

Task 4

A rectangle has a perimeter of 16 meters. Find the dimensions that maximize area and state the maximum area.

Answer: Length 4 meters, width 4 meters, maximum area 16 square meters.

Reason: Constraint: 2L + 2W = 16, so W = 8 - L. Objective: A = L(8 - L) = 8L - L squared. Vertex at L = -8 divided by (2 times -1) = 4. Then W = 4. Area = 16.

Check: Test L = 3 gives area 15; L = 5 gives area 15; both less than 16.

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Source rubric / reflection

- [ ] Defines variables with units.
- [ ] Writes the constraint equation correctly.
- [ ] Writes the objective formula correctly.
- [ ] Substitutes to get a single-variable quadratic.
- [ ] Computes the vertex accurately.
- [ ] Finds both side lengths and checks positivity.
- [ ] States the final answer with units and context.
- [ ] Uses a check such as a table or symmetry.

Reflection: What did you check, what did you change, and what will you try next?