Algebraic Fractions: Simplify, Add, and Solve with Confidence

Original tasks 1, 2, 3, 4 / complete keys

Task 1

Simplify (x^2 - 9)/(x^2 - x - 6).

Answer: (x + 3)/(x + 2), with x not equal to 3 and x not equal to -2.

Reason: Factor numerator as (x - 3)(x + 3) and denominator as (x - 3)(x + 2). Cancel (x - 3). Excluded values come from the original denominator: x = 3 and x = -2.

Check: Substitute x = 0 into original and simplified forms: original gives (-9)/(-6) = 1.5; simplified gives 3/2 = 1.5. Both match.

Task 2

Add 2/(x + 1) + 3/(x - 1).

Answer: (5x + 1)/((x + 1)(x - 1)), with x not equal to 1 and x not equal to -1.

Reason: The LCD is (x + 1)(x - 1). Rewrite as 2(x - 1)/LCD + 3(x + 1)/LCD. Combine numerators: 2x - 2 + 3x + 3 = 5x + 1.

Check: Let x = 2. Original: 2/3 + 3/1 = 2/3 + 3 = 11/3. Simplified: (10 + 1)/((3)(1)) = 11/3. Match.

Task 3

Solve 3/(x + 2) = 1/2.

Answer: x = 4.

Reason: Cross-multiply: 3 times 2 equals 1 times (x + 2), so 6 = x + 2, giving x = 4. The original denominator x + 2 is 6 at x = 4, not zero.

Check: Substitute x = 4: left side 3/6 = 1/2, right side 1/2. Match.

Task 4

Solve 1/(x - 1) + 1/(x + 1) = 2/(x^2 - 1).

Answer: No solution; x = 1 and x = -1 are excluded.

Reason: The LCD is (x - 1)(x + 1). Multiply every term: (x + 1) + (x - 1) = 2, so 2x = 2 and x = 1. But x = 1 makes the original denominator zero, so it is extraneous. x = -1 is also excluded. No valid solution remains.

Check: Substitute x = 1 into the original equation: 1/0 is undefined, confirming rejection. The cleared equation alone would falsely suggest x = 1.

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Source rubric / reflection

- [ ] Factors numerator and denominator completely before canceling.
- [ ] Cancels only identical factors, not terms.
- [ ] States excluded values from the original denominator.
- [ ] Uses a consistent, legible cancellation convention; alternative valid markings are accepted.
- [ ] Simplifies the final expression and checks arithmetic.
- [ ] For equations, verifies candidates against original denominators.

Reflection: What did you check, what did you change, and what will you try next?