Plotting Fourier Series Partial Sums by Adding Harmonic Terms

Original tasks 1, 2, 3 / complete keys

Task 1

A sawtooth-like series has first term (2/π) sin t and second nonzero term −(1/π) sin 2t. Compute S_1 and S_2 at t = π/2 and t = π/4. Use π ≈ 3.14 and decimals to two places.

Answer: At t = π/2: term 1 = 2/π ≈ 0.64; term 2 = −(1/π)(0) = 0; S_1 ≈ 0.64; S_2 ≈ 0.64. At t = π/4: sin(π/4) ≈ 0.71 and sin(π/2) = 1; term 1 ≈ (2/π)(0.71) ≈ 0.45; term 2 = −(1/π)(1) ≈ −0.32; S_1 ≈ 0.45; S_2 ≈ 0.45 − 0.32 = 0.13.

Reason: Evaluate each term at the sample point using sine values, then add term values. The second term is negative, so subtract its magnitude.

Check: Recompute with exact fractions: at π/4, term 1 = √2/π and term 2 = −1/π, so S_2 = (√2 − 1)/π ≈ 0.13. Compare with the decimal result.

Task 2

For a series with terms (4/π) sin t, (4/(3π)) sin 3t, and (4/(5π)) sin 5t, compute S_3 at t = π/2. Use π ≈ 3.14 and decimals to two places.

Answer: sin(π/2) = 1, sin(3π/2) = −1, sin(5π/2) = 1. Term 1 ≈ 1.27; term 2 ≈ −0.42; term 3 ≈ 0.25. S_3 ≈ 1.27 − 0.42 + 0.25 = 1.10.

Reason: Evaluate each harmonic at t = π/2, keeping signs, then sum the three term values in order.

Check: Add in a different order: 1.27 + 0.25 = 1.52; 1.52 − 0.42 = 1.10. The result matches.

Task 3

A partial sum table has S_1 = 0.80 at t = π/3. The second term is (1/π) cos 2t. Compute S_2 at t = π/3 using cos(2π/3) = −0.5 and π ≈ 3.14. Give two decimals.

Answer: Term 2 = (1/π)(−0.5) ≈ −0.16. S_2 ≈ 0.80 − 0.16 = 0.64.

Reason: The cosine value is negative, so the second term lowers the partial sum. Add the signed term value to S_1.

Check: Estimate: 1/π is about 0.32, half is about 0.16, and subtracting gives about 0.64. The exact check is 0.80 − 1/(2π).

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Original tasks 4 / complete keys

Task 4

You plot S_1 and S_2 for a square wave. At $t = \pi/6$, $S_1 \approx 0.64$ and $S_2 \approx 1.06$. At $t = \pi/2$, $S_1 \approx 1.27$ and $S_2 \approx 0.85$. Describe one visible difference between the two curves near these points and state a check you would perform.

Answer: Near $t = \pi/6$, S_2 is higher than S_1 by about 0.42; near $t = \pi/2$, S_2 is lower than S_1 by about 0.42. The S_2 curve shows an extra wiggle compared with the single hump of S_1. Check: recompute the second term at both points and confirm the sign and magnitude of the difference.

Reason: The second harmonic adds a signed value at each point. A positive addition raises the curve; a negative addition lowers it, creating a visible ripple.

Check: Compare the plotted points with the table values at $\pi/6$ and $\pi/2$. If the plotted difference does not match about ± 0.42, recheck the second-term evaluation.

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Source rubric / reflection

- [ ] Axes are labeled with consistent units and a clear vertical scale.
- [ ] Harmonic table lists n, exact coefficient, decimal coefficient, and term expression.
- [ ] Sample-point table shows each term separately and S_1, S_2, S_3.
- [ ] Plotted curves match table values at three checked points.
- [ ] Curves are labeled and distinguishable by line style or color.
- [ ] Comparison notes one visible change from adding each harmonic.
- [ ] Arithmetic is checkable and uses consistent units.
- [ ] Valid alternative conventions for labeling or line style are accepted if explained.

Reflection: What did you check, what did you change, and what will you try next?