Fractional Exponents and Radicals: Convert, Evaluate, Simplify

Complete worked example / separate answer key

Convert the fourth root of 81 cubed to fractional exponent form, evaluate it, and check the result.

Answer: The fourth root of 81 cubed is 81^(3/4). Since the fourth root of 81 is 3, raise 3 to the third power to get 27. Check: 27^4 = 531441 and 81^3 = 531441, so the value is consistent.

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Original tasks 1, 2, 3 / complete keys

Task 1

Convert the fifth root of x^3 to fractional exponent form and state one value of x for which the expression is defined.

Answer: x^(3/5). The expression is defined for all real x because the index 5 is odd, so odd roots of negative numbers are real. One valid choice is x = 32, giving 32^(3/5) = 8.

Reason: The index 5 becomes the denominator and the power 3 becomes the numerator. For x = 32, the fifth root is 2, and 2^3 = 8.

Check: Evaluate both forms at x = 32: the fifth root of 32^3 equals the fifth root of 32768, which is 8, matching 32^(3/5) = 8.

Task 2

Evaluate 64^(2/3) and 64^(-2/3). Show the order you used.

Answer: 64^(2/3) = 16 and 64^(-2/3) = 1/16.

Reason: The cube root of 64 is 4, and 4^2 = 16. The negative exponent gives the reciprocal, so 64^(-2/3) = 1/16.

Check: Check 16^3 = 4096 and 64^2 = 4096, so 64^(2/3) = 16 is consistent. Then 1/16 times 16 equals 1, confirming the reciprocal.

Task 3

Simplify the square root of 72 times the cube root of 72, leaving the answer in fractional exponent form.

Answer: 72^(5/6).

Reason: Convert to fractional exponents: 72^(1/2) times 72^(1/3). Add the exponents because the bases are the same: 1/2 + 1/3 = 3/6 + 2/6 = 5/6. So the product is 72^(5/6).

Check: Evaluate approximately with a test value: 72^(1/2) is about 8.485 and 72^(1/3) is about 4.160, whose product is about 35.3. Also 72^(5/6) is about 35.3, so the forms agree.

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Original tasks 4 / complete keys

Task 4

Simplify the cube root of 54 divided by the cube root of 2, and write the result as a radical and as a fractional exponent.

Answer: The cube root of 54 divided by the cube root of 2 equals the cube root of 27, which is 3. As a radical, the simplified result is 3. As a fractional exponent, the simplified result is 3^(1/1) = 3 (or simply 3); the intermediate form 27^(1/3) also equals 3.

Reason: Use the quotient rule for radicals: the cube root of 54 divided by the cube root of 2 equals the cube root of 54/2, which is the cube root of 27. Since 3^3 = 27, the cube root is 3. The fractional-exponent form of the simplified quotient is 27^(1/3), because the index 3 becomes the denominator and the power 1 becomes the numerator.

Check: Check by cubing the result: 3^3 = 27, and 54/2 = 27, so the quotient of the cube roots is 3. Also verify the fractional-exponent form: 27^(1/3) means the cube root of 27, which is 3, matching the radical result.

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Worked example, Task 1, Task 2, Task 3, Task 4 / Separate answer key with worked steps, reasoning, and checks

Full original answers, reasoning, and checks.

Worked example: The fourth root of 81 cubed is 81^(3/4). Since the fourth root of 81 is 3, raise 3 to the third power to get 27. Check: 27^4 = 531441 and 81^3 = 531441, so the value is consistent.

Exercise 1: x^(3/5). The expression is defined for all real x because the index 5 is odd, so odd roots of negative numbers are real. One valid choice is x = 32, giving 32^(3/5) = 8. Reasoning: the index 5 becomes the denominator and the power 3 becomes the numerator. For x = 32, the fifth root is 2, and 2^3 = 8. Check: evaluate both forms at x = 32: the fifth root of 32^3 equals the fifth root of 32768, which is 8, matching 32^(3/5) = 8.

Exercise 2: 64^(2/3) = 16 and 64^(-2/3) = 1/16. Reasoning: the cube root of 64 is 4, and 4^2 = 16. The negative exponent gives the reciprocal, so 64^(-2/3) = 1/16. Check: 16^3 = 4096 and 64^2 = 4096, so 64^(2/3) = 16 is consistent. Then 1/16 times 16 equals 1, confirming the reciprocal.

Exercise 3: 72^(5/6). Reasoning: convert to fractional exponents: 72^(1/2) times 72^(1/3). Add the exponents because the bases are the same: 1/2 + 1/3 = 3/6 + 2/6 = 5/6. So the product is 72^(5/6). Check: 72^(1/2) is about 8.485 and 72^(1/3) is about 4.160, whose product is about 35.3. Also 72^(5/6) is about 35.3, so the forms agree.

Exercise 4: The cube root of 54 divided by the cube root of 2 equals the cube root of 27, which is 3. As a radical, the simplified result is 3. As a fractional exponent, the simplified result is 3^(1/1) = 3 (or simply 3); the intermediate form 27^(1/3) also equals 3. Reasoning: use the quotient rule for radicals: the cube root of 54 divided by the cube root of 2 equals the cube root of 54/2, which is the cube root of 27. Since 3^3 = 27, the cube root is 3. The fractional-expo

Check: 3^3 = 27, and 54/2 = 27, so the quotient of the cube roots is 3. Also verify the fractional-exponent form: 27^(1/3) means the cube root of 27, which is 3, matching the radical result.

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Source rubric / reflection

- [ ] Conversion accuracy: the index and power are placed correctly in the fractional exponent form.
- [ ] Evaluation accuracy: the numeric result is correct and the order of root and power is valid.
- [ ] Simplification accuracy: exponents are combined only for like bases and perfect power factors are extracted correctly.
- [ ] Domain awareness: even roots of negative bases are identified as non-real, and odd roots of negative bases are handled correctly.
- [ ] Check method: the student provides a valid arithmetic or substitution check that supports the result.

Reflection: What did you check, what did you change, and what will you try next?