Graphing Linear-Quadratic Systems: Find Intersection Points

Complete worked example / separate answer key

Graph the system y = x + 1 and y = x squared - 1 on the same axes. Find the intersection points and verify them.

Answer: The intersection points are (-1, 0) and (2, 3). Check (-1, 0): line gives -1 + 1 = 0; parabola gives (-1) squared - 1 = 0. Check (2, 3): line gives 2 + 1 = 3; parabola gives 2 squared - 1 = 3. Both points satisfy both equations.

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Original tasks 1, 2, 3 / complete keys

Task 1

Graph y = x + 2 and y = x squared on the same axes. Find the intersection points and verify them.

Answer: The intersection points are (-1, 1) and (2, 4).

Reason: For the line, points (-1, 1) and (2, 4) lie on y = x + 2. For the parabola, (-1) squared = 1 and 2 squared = 4, so both points satisfy y = x squared. The graph shows two crossings.

Check: Substitute x = -1 into both equations: line gives 1, parabola gives 1. Substitute x = 2: line gives 4, parabola gives 4. Both match.

Task 2

Graph y = -x + 4 and y = x squared - 5x + 7 on the same axes. Find the intersection points and verify them.

Answer: The intersection points are (1, 3) and (3, 1).

Reason: Set the expressions equal: x squared - 5x + 7 = -x + 4. Add x and subtract 4 from both sides: x squared - 4x + 3 = 0. Factor: (x - 1)(x - 3) = 0, so x = 1 or x = 3. The line gives y = -1 + 4 = 3 at x = 1 and y = -3 + 4 = 1 at x = 3. The parabola gives 1 - 5 + 7 = 3 at x = 1 and 9 - 15 + 7 = 1 at x = 3. The graph shows two crossings, so there are two solutions.

Check: Substitute (1, 3): line -1 + 4 = 3; parabola 1 - 5 + 7 = 3. Substitute (3, 1): line -3 + 4 = 1; parabola 9 - 15 + 7 = 1. Both points satisfy both equations.

Task 3

Graph y = 2x - 3 and y = x squared - 4x + 5. Find the intersection points and verify them.

Answer: The intersection points are (2, 1) and (4, 5).

Reason: Set x squared - 4x + 5 = 2x - 3, so x squared - 6x + 8 = 0, which factors as (x - 2)(x - 4) = 0. Thus x = 2 or x = 4. The line gives y = 1 and y = 5. The parabola gives 4 - 8 + 5 = 1 and 16 - 16 + 5 = 5.

Check: Substitute (2, 1): line 2(2) - 3 = 1; parabola 4 - 8 + 5 = 1. Substitute (4, 5): line 8 - 3 = 5; parabola 16 - 16 + 5 = 5.

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Original tasks 4 / complete keys

Task 4

Graph y = x - 1 and y = x squared - 2x + 1 on the same axes. How many intersection points are there? Find them and verify.

Answer: There are two intersection points: (1, 0) and (2, 1).

Reason: Set the expressions equal: x squared - 2x + 1 = x - 1. Subtract x and add 1 to both sides: x squared - 3x + 2 = 0. Factor: (x - 1)(x - 2) = 0, so x = 1 or x = 2. The line gives y = 0 at x = 1 and y = 1 at x = 2. The parabola gives 1 - 2 + 1 = 0 at x = 1 and 4 - 4 + 1 = 1 at x = 2. The graph shows two crossings, so there are two solutions.

Check: Substitute (1, 0): line 1 - 1 = 0; parabola 1 - 2 + 1 = 0. Substitute (2, 1): line 2 - 1 = 1; parabola 4 - 4 + 1 = 1. Both points satisfy both equations.

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Worked example, Task 1, Task 2, Task 3, Task 4 / Answer Key: Intersection Points and Substitution Checks

Ordered pairs and substitution checks; note that graph reading may be approximate and should be verified algebraically.

Worked example: (-1, 0) and (2, 3). Check (-1, 0): line -1 + 1 = 0; parabola (-1) squared - 1 = 0. Check (2, 3): line 2 + 1 = 3; parabola 2 squared - 1 = 3.

Exercise 1: (-1, 1) and (2, 4). Check x = -1: line gives 1, parabola gives 1. Check x = 2: line gives 4, parabola gives 4.

Exercise 2: (1, 3) and (3, 1). Check (1, 3): line -1 + 4 = 3; parabola 1 - 5 + 7 = 3. Check (3, 1): line -3 + 4 = 1; parabola 9 - 15 + 7 = 1.

Exercise 3: (2, 1) and (4, 5). Check (2, 1): line 2(2) - 3 = 1; parabola 4 - 8 + 5 = 1. Check (4, 5): line 8 - 3 = 5; parabola 16 - 16 + 5 = 5.

Exercise 4: (1, 0) and (2, 1). Check (1, 0): line 1 - 1 = 0; parabola 1 - 2 + 1 = 0. Check (2, 1): line 2 - 1 = 1; parabola 4 - 4 + 1 = 1.

Note: graph reading may be approximate; if an intersection falls between grid lines, estimate to the nearest half unit and confirm by substitution.

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Source rubric / reflection

- [ ] Axes are labeled and tick spacing is equal or clearly stated.
- [ ] The line is drawn through two correctly computed points.
- [ ] The parabola uses at least five points around the vertex and is drawn as a smooth curve.
- [ ] All intersection points are marked and written as ordered pairs (x, y).
- [ ] Each intersection point is verified by substitution into both equations.
- [ ] The number of solutions (two, one, or none) is stated consistently with the graph.
- [ ] Handwriting and letter styles may vary; credit is given for clear labels and consistent scale, not for a single style.

Reflection: What did you check, what did you change, and what will you try next?