Partial Fractions: Decompose Rational Expressions Step by Step

Original tasks 1, 2, 3 / complete keys

Task 1

Decompose (5x+1)/((x-2)(x+3)) into partial fractions.

Answer: (11/5)/(x-2) + (14/5)/(x+3)

Reason: Set up A/(x-2) + B/(x+3). Multiply by (x-2)(x+3): 5x+1 = A(x+3) + B(x-2). Substitute x=2: 5(2)+1=11 and A(2+3)=5A, so A=11/5. Substitute x=-3: 5(-3)+1=-14 and B(-3-2)=-5B, so B=14/5. Thus the decomposition is (11/5)/(x-2) + (14/5)/(x+3). Check by combining: [11(x+3)+14(x-2)]/[5(x-2)(x+3)] = (11x+33+14x-28)/[5(x-2)(x+3)] = (25x+5)/[5(x-2)(x+3)] = (5x+1)/[(x-2)(x+3)], which matches the original.

Check: Combine (11/5)/(x-2) + (14/5)/(x+3) over the common denominator 5(x-2)(x+3); the numerator simplifies to 25x+5, which divided by 5 gives 5x+1, recovering the original expression.

Task 2

Decompose (2x+7)/((x+1)(x-4)) into partial fractions.

Answer: -1/(x+1) + 3/(x-4)

Reason: Set up A/(x+1) + B/(x-4). Multiply by (x+1)(x-4): 2x+7 = A(x-4) + B(x+1). Substitute x=-1: 2(-1)+7=5 and A(-1-4)=-5A, so -5A=5 and A=-1. Substitute x=4: 2(4)+7=15 and B(4+1)=5B, so B=3. Thus the decomposition is -1/(x+1) + 3/(x-4). Check by combining: [-1(x-4)+3(x+1)]/[(x+1)(x-4)] = (-x+4+3x+3)/[(x+1)(x-4)] = (2x+7)/[(x+1)(x-4)], which matches the original.

Check: Combine -1/(x+1) + 3/(x-4) over the common denominator (x+1)(x-4); the numerator simplifies to 2x+7, recovering the original expression.

Task 3

Decompose (4x+3)/((x-1)^2) into partial fractions.

Answer: 4/(x-1) + 7/(x-1)^2

Reason: Set A/(x-1)+B/(x-1)^2. Multiply: 4x+3=A(x-1)+B. x=1: 7=B. Then 4x+3=A(x-1)+7 => 4x-4=A(x-1) => A=4. Check: 4/(x-1)+7/(x-1)^2 = (4(x-1)+7)/(x-1)^2 = (4x-4+7)/(x-1)^2 = (4x+3)/(x-1)^2.

Check: Combine 4/(x-1)+7/(x-1)^2 to get (4x+3)/(x-1)^2.

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Original tasks 4 / complete keys

Task 4

Decompose (x+5)/((x+2)(x-1)^2) into partial fractions.

Answer: 1/3/(x+2) - 1/3/(x-1) + 2/(x-1)^2

Reason: Set A/(x+2)+B/(x-1)+C/(x-1)^2. Multiply: x+5=A(x-1)^2+B(x+2)(x-1)+C(x+2). x=-2: 3=9A => A=1/3. x=1: 6=3C => C=2. Match x^2: 0=A+B => B=-1/3. Check: (1/3)/(x+2)+(-1/3)/(x-1)+2/(x-1)^2. Combine: common denom 3(x+2)(x-1)^2. Numerator: (x-1)^2 - (x+2)(x-1) + 6(x+2) = (x^2-2x+1) - (x^2+x-2) +6x+12 = x^2-2x+1 -x^2-x+2+6x+12 =3x+15=3(x+5). Divide by 3 gives (x+5)/den.

Check: Combine the three fractions and simplify to (x+5)/((x+2)(x-1)^2).

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Source rubric / reflection

- [ ] Correctly identifies proper rational expression and factors denominator.
- [ ] Sets up partial fractions with correct number and form of terms, including repeated factors.
- [ ] Clears denominators correctly and forms a valid polynomial identity.
- [ ] Solves for constants accurately using substitution or coefficient matching.
- [ ] Writes final decomposition with constants in simplest form.
- [ ] Verifies by recombining and simplifying to original expression.
- [ ] Handwriting: x-height from baseline to midline; letter styles may vary but algebraic symbols are clear and unambiguous.

Reflection: What did you check, what did you change, and what will you try next?