# Polygonal Numbers: Build Sequences and Derive nth-Term Formulas

Original tasks 1, 2, 3 / complete keys

Task 1

A dot diagram shows triangular numbers. Term 1 is one dot. Term 2 adds a ring of 2 dots. Term 3 adds a ring of 3 dots. Term 4 adds a ring of 4 dots. Build the first five terms, compute differences, and derive the nth-term formula.

Answer: Terms: 1, 3, 6, 10, 15. Formula: Tn = n(n+1)/2.

Reason: First differences are 2, 3, 4, 5; second differences are 1, 1, 1. So 2a = 1, a = 1/2. Using T1 = 1 and T2 = 3 gives b = 1 and c = 0.

Check: Substitute n = 5 into n(n+1)/2 to get 15, matching the counted fifth term.

Task 2

A sequence begins 1, 6, 15, 28, 45. A dot diagram for term 3 shows a central dot, a ring of 5 dots, and a ring of 9 dots. Identify the polygon family, compute second differences, and derive the formula.

Answer: Hexagonal numbers. Formula: Hn = 2n^2 - n.

Reason: First differences are 5, 9, 13, 17; second differences are 4, 4, 4. So 2a = 4, a = 2. Using H1 = 1 and H2 = 6 gives b = -1 and c = 0.

Check: Substitute n = 4 into 2n^2 - n to get 28, matching the fourth term.

Task 3

Square numbers have terms 1, 4, 9, 16, 25. A dot diagram for term 4 is a 4-by-4 array. Derive the nth-term formula using differences and verify for n = 6.

Answer: Sn = n^2. For n = 6, S6 = 36.

Reason: First differences are 3, 5, 7, 9; second differences are 2, 2, 2. So 2a = 2, a = 1. Using S1 = 1 and S2 = 4 gives b = 0 and c = 0.

Check: A 6-by-6 array has 36 dots, matching n^2.

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Original tasks 4 / complete keys

Task 4

A sequence has constant second difference 5 and begins 1, 7, 18, 34, 55. Identify the polygon family, derive the formula, and verify for n = 5.

Answer: Heptagonal numbers (s = 7). Formula: (5n^2 - 3n)/2. For n = 5, (125 - 15)/2 = 55.

Reason: Second difference 5 means 2a = 5, a = 5/2. Using first term 1: 5/2 + b + c = 1. Using second term 7: 10 + 2b + c = 7. Subtracting gives b = -3/2 and c = 0.

Check: Substitute n = 4 into (5n^2 - 3n)/2 to get (80 - 12)/2 = 34, matching the fourth term.

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Source rubric / reflection

- [ ] Dot diagrams correctly show nested rings with counts matching the stated terms.
- [ ] Table lists at least five terms with n = 1 through 5.
- [ ] First differences are computed correctly and increase by a constant amount.
- [ ] Second difference is identified as constant and equal to s - 2.
- [ ] Value of a is correctly set to half the second difference.
- [ ] Equations from n = 1 and n = 2 are solved correctly for b and c.
- [ ] Closed-form formula is stated with n defined as the term number.
- [ ] Formula is verified against at least one counted term beyond n = 2.
- [ ] Handwriting and diagram conventions are consistent and legible; alternative valid layouts are accepted if counts are correct.

Reflection: What did you check, what did you change, and what will you try next?