Polygonal Numbers: Build Sequences and Derive nth-Term Formulas

Worked example / complete source key

Build the first five pentagonal numbers from dot diagrams, compute first and second differences, and derive the nth-term formula. Verify the formula for n = 5.

Answer: Terms: 1, 5, 12, 22, 35. First differences: 4, 7, 10, 13. Second differences: 3, 3, 3. Constant second difference 3 means 2a = 3, so a = 3/2. Using P1 = 1: 3/2 + b + c = 1. Using P2 = 5: 6 + 2b + c = 5. Subtracting: b = 1/2, then c = 0. Formula: Pn = (3n^2 - n)/2. Check n = 5: (75 - 5)/2 = 35, matching the counted term.