Draw the Star of David: Overlapping Triangle Construction

Original tasks 1, 2 / complete keys

Task 1

A student draws two centered equilateral triangles with side length 4 cm. The top apex is 3.5 cm above center C. Where should the bottom apex be for symmetry, and what is the correct center-to-apex distance for side 4 cm? Distinguish it from the triangle altitude.

Answer: Mirror the second triangle's bottom apex 3.5 cm below C to match the student's first apex, but that is not the ideal centered placement. For side 4 cm, $r=4/\sqrt{3}\approx2.31$ cm; place the top and bottom apexes about 2.31 cm above and below C. The triangle altitude from its base to its apex is a different measure: $4\sqrt{3}/2\approx3.46$ cm.

Reason: The two triangle apexes are symmetric about C, so a 3.5 cm top offset would mirror to 3.5 cm below. However, the Star of David is centered at the triangles' common centroid; the vertex radius is $s/\sqrt{3}$, not the triangle altitude. For $s=4$ cm, $r\approx2.31$ cm, while $h\approx3.46$ cm.

Check: Compute $r=4/\sqrt{3}\approx2.31$ cm and verify the top and bottom apexes are at $+r$ and $-r$ from C. Separately compute $h=4\sqrt{3}/2\approx3.46$ cm; do not use h as the center-to-apex distance.

Task 2

For two centered equilateral triangles with side length 8.66 cm, a student measures the six outer-point distances from center C as 5.0, 5.1, 4.9, 5.0, 5.2, and 5.0 cm. Using a chosen tolerance of ±0.2 cm, do the measurements pass the radius check?

Answer: Yes. The ideal circumradius is $8.66/\sqrt{3}\approx5.00$ cm. The measured distances differ from 5.00 cm by 0, 0.1, 0.1, 0, 0.2, and 0 cm, all within the stated ±0.2 cm tolerance.

Reason: The centered outer vertices lie at the circumradius $s/\sqrt{3}$. Here the chosen side gives a radius of about 5.00 cm, and the largest absolute measurement deviation is 0.20 cm. The tolerance is explicitly an exercise choice.

Check: Compute $8.66/\sqrt{3}\approx5.00$ cm and compare each reading with 5.00 cm; confirm every absolute deviation is no more than 0.2 cm.

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Original tasks 3, 4 / complete keys

Task 3

A student's first triangle has side length 5 cm and altitude 4.3 cm. The second triangle has side length 5 cm but altitude 4.0 cm. What is the likely mistake, and how can it be fixed?

Answer: The second triangle's altitude is incorrect. For a 5 cm side the altitude should be about 4.33 cm (5 times 0.866 is about 4.33); the first triangle's 4.3 cm is acceptable rounding, but 4.0 cm is too short. Redraw the second triangle with altitude about 4.33 cm.

Reason: For an equilateral triangle, altitude is determined by side length: altitude equals side length times about 0.866. With a 5 cm side, the altitude should be about 4.33 cm. A 4.0 cm altitude with a 5 cm side indicates a construction error in the second triangle.

Check: Measure all three sides of the second triangle; they should be equal, and its altitude should match the first triangle's altitude (both about 4.33 cm for a 5 cm side).

Task 4

Describe how to check that the second triangle is rotated exactly 180 degrees from the first, using only a ruler and the drawn lines.

Answer: Check that the top vertex of the first triangle and the bottom vertex of the second triangle lie on the same vertical midline, with the center point exactly between them. Also check that the bases are parallel and on opposite sides of the center.

Reason: A 180-degree rotation maps the top vertex to the bottom vertex along the same line through the center, and maps the base to a parallel line on the opposite side.

Check: Measure the distance from the center to the top vertex and to the bottom vertex; they should be equal, and both vertices should be on the midline.

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Source rubric / reflection

- [ ] Both triangles are equilateral within drawing accuracy (side lengths equal).
- [ ] The two triangles share the same center point.
- [ ] The second triangle is rotated 180 degrees relative to the first.
- [ ] Six outer points are present and roughly equally spaced around the center.
- [ ] Symmetry check shows equal distances from center to outer points.
- [ ] Line style and point sharpness may vary; structural symmetry is the primary criterion.

Reflection: What did you check, what did you change, and what will you try next?